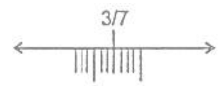
1.2 RATIONAL NUMBER

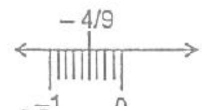
REPRESENTATIO OF RATIONAL NUMBER ON A REAL NUMBER LINE

(i) 3/7 Divide a unit into 7 equal parts.



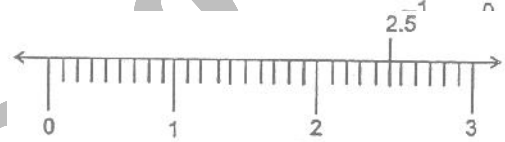
(ii) $\frac{13}{7}$

(iii) $-\frac{4}{9}$

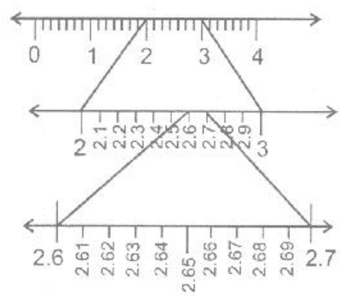


(a) Decimal Number (Terminating) :

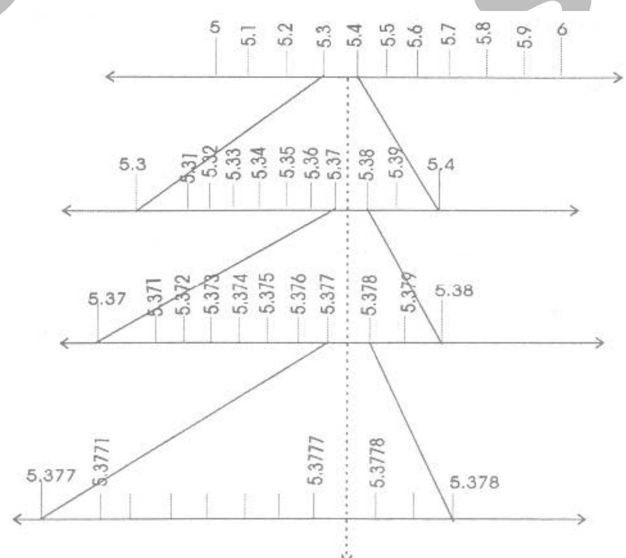
(i) 2.5



(ii) 2.65 (process of magnification)



Ex.3 Visualize the representation of $5.3\overline{7}$ on the number line upto 5 decimal place. i.e. 5.37777.



(b) Find Rational Numbers Between Two Integral Numbers :

Ex.4 Find 4 rational numbers between 2 and 3.

Sol. **Steps :**

(i) Write 2 and 3 multipling in N^r and D^r with (4+1).

(ii) i.e. 2 $\frac{2\times(4+1)}{(4+1)} = \frac{1}{5}$ & $3 = \frac{3\times(4+1)}{(4+1)} = \frac{15}{5}$

(iii) So, the four required numbers are $\frac{11}{5}, \frac{12}{5}, \frac{13}{5}, \frac{14}{5}$.

Ex.5 Find three rational no's between a and b (a < b).

Sol.

$a < b$

$\Rightarrow a + a < b + a$

$\Rightarrow 2a < a + b$

$\Rightarrow a < \frac{a+b}{2}$

Again, $a < b$

$\Rightarrow a + b < b + b.$

$\Rightarrow a + b < 2b$

$\Rightarrow \frac{a+b}{2} < b.$

$\therefore a < \frac{a+b}{2} < b.$

i.e. $\frac{a+b}{2}$ lies between a and b.

Hence 1st rational number between a and b is $\frac{a+b}{2}$.

For next rational number

$$\frac{a+\frac{a+b}{2}}{2} = \frac{\frac{2a+a+b}{2}}{2} = \frac{3a+b}{4} \quad \therefore \quad a < \frac{3a+b}{4} < \frac{a+b}{2} < b.$$

Next, $$\frac{\frac{a+b}{2}+b}{2} = \frac{a+b+2b}{2\times2} = \frac{a+3b}{4}$$

$\therefore \quad a < \frac{3a+b}{4} < \frac{a+b}{2} < \frac{a+3b}{4} < b$, and continues like this.

Ex.6 Find 3 rational numbers between $\frac{1}{3}$ & $\frac{1}{2}$.

Sol. 1st Method $$\frac{\frac{1}{3}+\frac{1}{2}}{2} = \frac{\frac{2+3}{6}}{2} = \frac{5}{12} \quad \therefore \quad \frac{1}{3}, \frac{5}{12}, \frac{1}{2}$$

$$= \frac{\frac{1}{3}+\frac{5}{12}}{2} = \frac{\frac{4+5}{12}}{2} = \frac{9}{24} \quad \therefore \quad \frac{1}{3}, \frac{9}{24}, \frac{5}{12}, \frac{1}{2}$$

$$= \frac{\frac{5}{12}+\frac{1}{2}}{2} = \frac{\frac{5}{12}+\frac{6}{12}}{2} = \frac{11}{24} \qquad \therefore \qquad \frac{1}{3},\frac{9}{24},\frac{5}{12},\frac{11}{24},\frac{1}{2}.$$

Verify : $\frac{8}{24} < \frac{9}{24} < \frac{10}{24} < \frac{11}{24} < \frac{12}{24}.\left(\text{as}\frac{8}{24} = \frac{1}{3} \;\&\; \frac{1}{2}\right)$

2^{nd} Method : Find n rational numbers between a and b (a < b).

(i) Find d = $\frac{b-a}{n+1}$.

(ii) 1st rational number will be a + d.
2nd rational number will be a + 2d.
3rd rational number will be a + 3d and so on....
nth rational number is a + nd.

Ex.7 Find 5 rational number between $\frac{3}{5}$ and $\frac{4}{5}$

Here, $a = \frac{3}{5}, b = \frac{4}{5} d = \frac{b-a}{n+1} = \frac{\frac{4}{5}-\frac{3}{5}}{5+1} = \frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$.

$1^{st} = a + b = \frac{3}{5} + \frac{1}{30} = \frac{19}{20}$, $\qquad 2^{nd} = a + 2d = \frac{3}{5} + \frac{2}{30}$,

$3^{rd} = a + 3d = \frac{3}{5} + \frac{3}{30} = \frac{21}{30}$, $\qquad 4^{th} = a + 4d = \frac{3}{5} + \frac{4}{30} = \frac{22}{30}$,

$5^{th} = a + 5d = \frac{3}{5} + \frac{5}{30} = \frac{23}{30}$.

RATIONAL NUMBER IN DECIMAL REPRESENTATION

(a) Terminating Decimal :

In this a finite number of digit occurs after decimal i.e. $\frac{1}{2}$ = 0.5, 0.6875, 0.15 etc.

(b) Non-Terminating and Repeating (Recurring Decimal) :

In this a set of digits or a digit is repeated continuously.

Ex.8 $\frac{2}{3} = 0.6666 ------- = 0.\overline{6}$.

Ex.9 $\frac{5}{11} = 0.454545 ------- = 0.\overline{45}$.

PROPERTIES OF RATIONAL NUMBER

If a,b,c are three rational numbers.

(i) Commutative property of addition. a + b = b + a

(ii) Associative property of addition (a+b)+c = a+(b+c)

(iii) Additive inverse a + (-a) = 0

0 is identity element, -a is called additive inverse of a.

(iv) Commutative property of multiplications a.b. = b.a.

(v) Associative property of multiplication (a.b).c = a.(b.c)

(vi) Multiplicative inverse $(a)\times\left(\frac{1}{a}\right) = 1$

1 is called multiplicative identity and $\frac{1}{a}$ is called multiplicative inverse of a or reciprocal of a.

(vii) Distributive property a.(b+c) = a.b + a.c

Ex.9 Prove that $\sqrt{3}-\sqrt{2}$ is an irrational number

Sol. Let $\sqrt{3}-\sqrt{2}=r$ where r be a rational number

Squaring both sides

$\Rightarrow \left(\sqrt{3}-\sqrt{2}\right)^2 = r^2$

$\Rightarrow 3 + 2 - 2\sqrt{6} = r^2$

$\Rightarrow 5 - 2\sqrt{6} = r^2$

Here, $5 - 2\sqrt{6}$ is an irrational number but r^2 is a rational number

$\therefore$ L.H.S. $\neq$ R.H.S.

Hence it contradicts our assumption that $\sqrt{3}-\sqrt{2}$ is a rational number.

(b) Irrational Number in Decimal Form :

$\sqrt{2}$ = 1.414213 ...... i.e. it is not-recurring as well as non-terminating.

$\sqrt{3}$ = 1.732050807 ...... i.e. it is non-recurring as well as non-terminating.

Ex.10 Insert an irrational number between 2 and 3.

Sol. $\sqrt{2\times 3}=\sqrt{6}$

Ex.11 Find two irrational number between 2 and 2.5.

Sol. 1st **Method :** $\sqrt{2\times 2.5}=\sqrt{5}$

Since there is no rational number whose square is 5. So $\sqrt{5}$ is irrational..

Also $\sqrt{2\times\sqrt{5}}$ is a irrational number.

2nd Method : 2.101001000100001.... is between 2 and 5 and it is non-recurring as well as non-terminating.

Also, 2.201001000100001......... and so on.

Ex.12 Find two irrational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol. **1st Method :** $\sqrt{\sqrt{2}\times\sqrt{3}}=\sqrt{\sqrt{6}}=\sqrt[4]{6}$

Irrational number between $\sqrt{2}$ and $\sqrt[4]{6}$

$\sqrt{\sqrt{2}\times\sqrt[4]{6}}=\sqrt[4]{2}\times\sqrt[8]{6}$

2nd Method : As $\sqrt{2}$ = 1.414213562 ...... and $\sqrt{3}$ = 1.732050808......

As , $\sqrt{3} > \sqrt{2}$ and $\sqrt{2}$ has 4 in the 1st place of decimal while $\sqrt{3}$ has 7 is the 1st place of decimal.

$\therefore$ 1.501001000100001......., 1.601001000100001...... etc. are in between $\sqrt{2}$ and $\sqrt{3}$

Ex.13 Find two irrational number between 0.12 and 0.13.

Sol. 0.1201001000100001......., 0.12101001000100001 ....etc.

Ex.14 Find two irrational number between 0.3030030003..... and 0.3010010001 ....

Sol. 0.302020020002...... 0.302030030003.... etc.

Ex.15 Find two rational number between 0.2323323332..... and 0.252552555255552.......

Sol. 1st place is same 2.

2nd place is 3 & 5.

3rd place is 2 in both.

4th place is 3 & 5.

Let a number = 0.25, it falls between the two irrational number.

Also a number = 0.2525 an so on.