1.3 IRRATIONAL NUMBER

GEOMETRICAL REPRESENTATION OF REAL NUMBERS

To represent any real number of number line we follows the following steps :

STEP I : Obtain the positive real number x (say).

STEP II : Draw a line and mark a point A on it.

STEP III : Mark a point B on the line such that AB = x units.

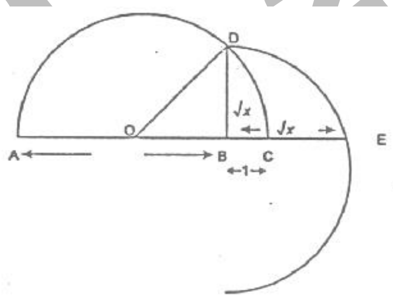
STEP IV : From point B mark a distance of 1 unit and mark the new point as C.

STEP V : Find the mid - point of AC and mark the point as O.

STEP VI : Draw a circle with centre O and radius OC.

STEP VII : Draw a line perpendicular to AC passing through B and intersecting the semi circle at D.

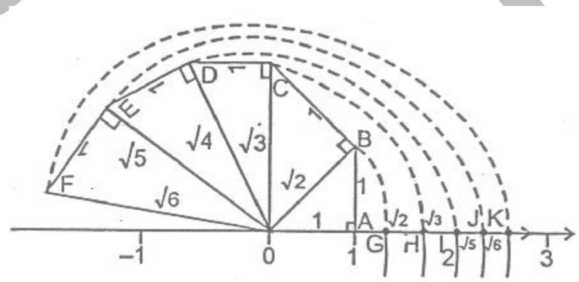
Length BD is equal to $\sqrt{x}$.



(c) Irrational Number on a Number Line :

Ex.16 Plot $\sqrt{2}, \sqrt{3}, \sqrt{5}\sqrt{6}$ on a number line.

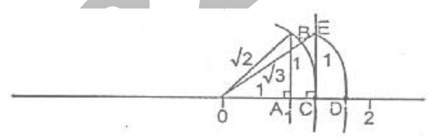
Sol.



Another Method for :

(i) Plot $\sqrt{2}, \sqrt{3}$

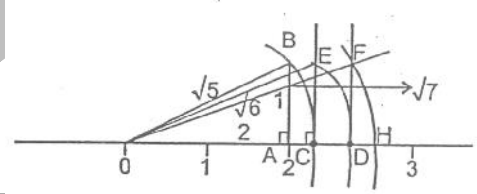
So, OC = $\sqrt{2}$ and OD = $\sqrt{3}$



(ii) Plot $\sqrt{5}, \sqrt{6}, \sqrt{7}\sqrt{8}$

OC = $\sqrt{5}$

OD = $\sqrt{6}$ OH = $\sqrt{7}$ ....



(d) Properties of Irrational Number :

(i) Negative of an irrational number is an irrational number e.g. $-\sqrt{3}-\sqrt[4]{5}$ are irrational.

(ii) Sum and difference of a rational and an irrational number is always an irrational number.

(iii) Sum and difference of two irrational numbers is either rational or irrational number.

(iv) Product of a non-zero rational number with an irrational number is either rational or irrationals

(v) Product of an irrational with a irrational is not always irrational.

Ex.17 Two number's are 2 and $\sqrt{3}$, then

Sum = 2 + $\sqrt{3}$, is an irrational number.

Difference = 2 - $\sqrt{3}$, is an irrational number.

Also $\sqrt{3}-2$ is an irrational number.

Ex.18 Two number's are 4 and $\sqrt[3]{3}$, then

Sum = 4 + $\sqrt[3]{3}$, is an irrational number.

Difference = 4 - $\sqrt[3]{3}$, is an irrational number.

Ex.19 Two irrational numbers are $\sqrt{3},-\sqrt{3}$, then

Sum = $\sqrt{3}+\left(-\sqrt{3}\right)=0$ which is rational.

Difference = $\sqrt{3}-\left(-\sqrt{3}\right)=2\sqrt{3}$, which is irrational.

Ex.20 Two irrational numbers are 2 + $\sqrt{3}$ and 2 - $\sqrt{3}$, then

Sum = $\left(2+\sqrt{3}\right)+\left(2-\sqrt{3}\right)=4$, a rational number

Two irrational numbers are $\sqrt{3}+3m\sqrt{3}-3$

Difference = $\sqrt{3}+3-\sqrt{3}+3=6$, a rational number

Ex.21 Two irrational numbers are $\sqrt{3}-\sqrt{2},\sqrt{3}+\sqrt{2}$, then

Sum = $\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}=2\sqrt{3}$, an irrational

Ex.22 2 is a rational number and $\sqrt{3}$ is an irrational.

2 × $\sqrt{3}=2\sqrt{3}$, an irrational.

Ex.23 0 a rational and $\sqrt{3}$ an irrational.

0 × $\sqrt{3}$ = 0, a rational.

Ex.24 $\frac{4}{3}\times\sqrt{3}=\frac{4}{3}\sqrt{3}=\frac{4}{\sqrt{3}}$ is an irrational.

Ex.25 $\sqrt{3}\times\sqrt{3}=\sqrt{3\times3}=\sqrt{9}=3$ a rational number.

Ex.26 $2\sqrt{3}\times 3\sqrt{2} = 2\times 3\sqrt{3\times 2} = 6\sqrt{6}$ and irrational number.

Ex.27 $\sqrt[3]{3}\times\sqrt[3]{3^2} = \sqrt[3]{3\times 3^2} = \sqrt[3]{3^3} = 3$ a rational number.

Ex.28 $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) = (2)^2 - \left(\sqrt{3}\right)^2 = 4-3 = 1$ a rational number.

Ex.29 $\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right) = \left(2+\sqrt{3}\right)^2$

$= (2)^2 + \left(\sqrt{3}\right)^2 + 2(2)\times\left(\sqrt{3}\right)$

$= 4+3+4\sqrt{3}$

$= 7 + 4\sqrt{3}$ an irrational number

NOTE :

(i) $\sqrt{-2} \neq -\sqrt{2}$, it is not a irrational number.

(ii) $\sqrt{-2}\times\sqrt{-3} \neq \left(\sqrt{-2\times -3} = \sqrt{6}\right)$

Instead $\sqrt{-2}, \sqrt{-3}$ are called Imaginary numbers.

$\sqrt{-2} = i\sqrt{2}$, where i (= iota) = $\sqrt{-1}$

$\therefore$ (A) $i^2 = -1$

(B) $i^3 = i^2 \times i = (-1)\times i = -i$

(C) $i^4 = i^2 \times i^2 = (-1)\times(-1) = 1$

(iii) Numbers of the type (a + ib) are called complex numbers where (a, b) $\in$ R.e.g. 2 + 3i, -2 + 4i, -3i, 11 - 4i, are complex numbers.

EXERCISE 1.2

4. Examine whether the following numbers are rational or irrational :

(i) $\left(2-\sqrt{3}\right)^2$ (ii) $\left(\sqrt{2}+\sqrt{3}\right)^2$ (iii) $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$ (iv) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

5. Represent $\sqrt{8.3}$ on the number line.

6. Represent $(2+\sqrt{3})$ on the number line.

7. Prove that $\left(\sqrt{2}+\sqrt{5}\right)$ is an irrational number.

8. Prove that $\sqrt{7}$ is not a rational number.

9. Prove that $(2+\sqrt{2})$ is an irrational number.

10. Multiply $\sqrt{27a^3b^2c^4}\times\sqrt[3]{128a^7b^9c^2}\times\sqrt[6]{729ab^{12}c^2}$.

11. Express the following in the form of p/q.

(i) $0.\overline{3}$ (ii) $0.\overline{37}$ (iii) $0.\overline{54}$ (iv) $0.\overline{05}$ (v) $1.\overline{3}$ (vi) $0.\overline{621}$

12. Simplify : $0.\overline{4}+.01\overline{8}$