1.4 SURDS

Any irrational number of the form $\sqrt[n]{a}$ is given a special name surd. Where 'a' is called radicand, it should always be a rational number. Also the symbol $\sqrt[n]{\ }$ is called the radical sign and the index n is called order of the surd.

$\sqrt[n]{a}$ is read as 'n[th] **root a'** and can also be written as $a^{\frac{1}{n}}$.

(a) Some Identical Surds :

(i) $\sqrt[3]{4}$ is a surd as radicand is a rational number.

Similar examples $\sqrt[3]{5}, \sqrt[4]{12}, \sqrt[5]{7}, \sqrt{12}$,.........

(i) $2\sqrt{3}$ is a surd (as surd + rational number will give a surd)

Similar examples $\sqrt{3}+1, \sqrt[3]{3}+1$,....

(iii) $\sqrt{7-4\sqrt{3}}$ is a surd as 7 - $4\sqrt{3}$ is a perfect square of $(2-\sqrt{3})$

Similar examples $\sqrt{7+4\sqrt{3}}, \sqrt{9-4\sqrt{5}}, \sqrt{9+4\sqrt{5}}$,.........

(i) $\sqrt[3]{\sqrt{3}}$ is a surd as $\sqrt[3]{\sqrt{3}} = \left(3^{\frac{1}{2}}\right)^{\frac{1}{3}} = 3^{\frac{1}{6}} = \sqrt[6]{3}$

Similar examples $\sqrt[3]{\sqrt[3]{5}}, \sqrt[4]{\sqrt[5]{6}}$,....... ...

(b) Some Expression are not Surds :

(i) $\sqrt[3]{8}$ because $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$, which is a rational number.

(ii) $\sqrt{2+\sqrt{3}}$ because $2+\sqrt{3}$ is not a perfect square.

(iii) $\sqrt[3]{1+\sqrt{3}}$ because radicand is an irrational number.

LAWS OF SURDS

(i) $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n} = a$

e.g. (A) $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$ (B) $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

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(ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ [Here order should be same]

e.g. (A) $\sqrt[3]{2} \times \sqrt[3]{6} = \sqrt[3]{2 \times 6} = \sqrt[3]{12}$

but, $\sqrt[3]{3} \times \sqrt[4]{6} \neq \sqrt{3 \times 6}$ [Because order is not same]

1st make their order same and then you can multiply.

(iii) $\sqrt[n]{a} + \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$

(iv) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a} = \sqrt[m]{\sqrt[n]{a}}$ e.g. = $\sqrt{\sqrt{\sqrt{2}}} = \sqrt[8]{8}$

(v) $\sqrt[n]{a} = \sqrt[n \times p]{a^p}$ [Important for changing order of surds]

or, $\sqrt[n]{a^m} = \sqrt[n \times p]{a^{m \times p}}$

e.g. $\sqrt[3]{6^2}$ make its order 6, then $\sqrt[3]{6^2} = \sqrt[3 \times 2]{6^{2 \times 2}} = \sqrt[6]{6^4}$.

e.g. $\sqrt[3]{6}$ make its order 15, then $\sqrt[3]{6} = \sqrt[3 \times 5]{6^{1 \times 5}} = \sqrt[15]{6^5}$.

OPERATION OF SURDS

(a) Addition and Subtraction of Surds :

Addition and subtraction of surds are possible only when order and radicand are same i.e. only for surds.

Ex.1 Simplify

(i) $\sqrt{6} - \sqrt{216} + \sqrt{96} = 15\sqrt{6} - \sqrt{6^2 \times 6} + \sqrt{16 \times 6}$ [Bring surd in simples form]

$= 15\sqrt{6} - 6\sqrt{6} + 4\sqrt{6}$

$= (15 - 6 + 4)\sqrt{6}$

$= 13\sqrt{6}$ **Ans.**

(ii) $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$ $= 5\sqrt[3]{125 \times 2} + 7\sqrt[3]{8 \times 2} - 14\sqrt[3]{27 \times 2}$

$= 5 \times 5\sqrt[3]{2} + 7 \times 2\sqrt[3]{2} - 14 \times 3 \times \sqrt[3]{2}$

$= (25 + 14 - 42)\sqrt[3]{2}$

$= -3\sqrt[3]{2}$ **Ans.**

(ii) $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$ $= 5\sqrt[3]{125 \times 2} + 7\sqrt[3]{8 \times 2} - 14\sqrt[3]{27 \times 2}$

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$= (25 + 14 - 42)\sqrt[3]{2}$

$= -3\sqrt[3]{2}$ **Ans.**

(iii) $4\sqrt{3} + 3\sqrt{48} - \frac{5}{2}\sqrt{\frac{1}{3}} = 4\sqrt{3} + 3\sqrt{16 \times 3} - \frac{5}{2}\sqrt{\frac{1 \times 3}{3 \times 3}}$

$= 4\sqrt{3} + 3 \times 4\sqrt{3} - \frac{5}{2} \times \frac{1}{3}\sqrt{3}$

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$$= 4\sqrt{3} + 12\sqrt{3} - \frac{5}{6}\sqrt{3}$$

$$= \left(4 + 12 - \frac{5}{6}\right)\sqrt{3}$$

$$= \frac{91}{6}\sqrt{3}$$ **Ans.**

(b) Multiplication and Division of Surds :

Ex.2 (i) $\sqrt[3]{4} \times \sqrt[3]{22} = \sqrt[3]{4 \times 22} = \sqrt[3]{2^3 \times 11} = 2\sqrt[3]{11}$

(i) $\sqrt[3]{2} \times \sqrt[4]{3} = \sqrt[12]{2^4} \times \sqrt[12]{3^3} = \sqrt[12]{2^4 \times 3^3} = \sqrt[12]{16 \times 27} = \sqrt[12]{432}$

Ex.3 Simplify $\sqrt{8a^5b} \times \sqrt[3]{4a^2b^2}$

Hint : $\sqrt[6]{8^3a^{15}b^3} \times \sqrt[6]{4^2a^4b^4} = \sqrt[6]{2^{13}a^{19}b^7} = \sqrt[6]{2ab}$. **Ans.**

Ex.4 Divide $\sqrt{24} \div \sqrt[3]{200} = \dfrac{\sqrt{24}}{\sqrt[3]{200}} = \dfrac{\sqrt[6]{(24)^3}}{\sqrt[6]{(200)^2}} = \sqrt[6]{\dfrac{216}{625}}$ **Ans..**

(c) Comparison of Surds :

It is clear that if x > y > 0 and n > 1 is a positive integer then $\sqrt[n]{x} > \sqrt[n]{y}$.

Ex.5 $\sqrt[3]{16} > \sqrt[3]{12}, \sqrt[5]{35} > \sqrt[5]{25}$ and so on.

Ex.6 Which is greater is each of the following :

(i) $\sqrt[3]{16}$ and $\sqrt[5]{8}$

L.C.M. of 3 and 5 15.

$\sqrt[3]{6} = \sqrt[3\times5]{6^5} = \sqrt[15]{7776}$

$\sqrt[5]{8} = \sqrt[3\times5]{8^5} = \sqrt[15]{512}$

$\therefore \quad \sqrt[75]{7776} > \sqrt[15]{512}$

$\Rightarrow \quad \sqrt[3]{6} > \sqrt[5]{8}$

(ii) $\sqrt{\frac{1}{2}}$ and $\sqrt[3]{\frac{1}{3}}$

L.C.M. of 2 and 3 is 6.

$\sqrt[6]{\left(\frac{1}{2}\right)^3}$ and $\sqrt[3]{\left(\frac{1}{3}\right)^2}$

$\sqrt[6]{\frac{1}{8}}$ and $\sqrt[6]{\frac{1}{9}}$ $\left[\text{As } 8 < 9 \therefore \frac{1}{8} > \frac{1}{9}\right]$

so, $\sqrt[6]{\frac{1}{8}} > \sqrt[6]{\frac{1}{9}}$

$\Rightarrow \quad \sqrt{\frac{1}{2}} > \sqrt[3]{\frac{1}{3}}$

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Ex.7 Arrange $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[4]{5}$ is ascending order.

Sol. L.C.M. of 2, 3, 4 is 12.

$\therefore \quad \sqrt{2} = \sqrt[2\times6]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = \sqrt[3\times4]{3^4} = \sqrt[12]{81}$

$\sqrt[4]{5} = \sqrt[4\times3]{5^3} = \sqrt[12]{125}$

As, $64 < 81 < 125$.

$\therefore \quad \sqrt[12]{64} < \sqrt[12]{81} < \sqrt[12]{125}$

$\Rightarrow \quad \sqrt{2} < \sqrt[3]{3} < \sqrt[4]{5}$

Ex.8 Which is greater $\sqrt{7} - \sqrt{3}$ or $\sqrt{5} - 1$?

Sol. $\sqrt{7} - \sqrt{3} = \frac{(\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3})}{(\sqrt{7} + \sqrt{3})} = \frac{7-3}{\sqrt{7} + \sqrt{3}} = \frac{4}{\sqrt{7} + \sqrt{3}}$

And, $\sqrt{5} - 1 = \frac{(\sqrt{5} - 1)(\sqrt{5} + 1)}{(\sqrt{5} + 1)} = \frac{5-1}{\sqrt{5} + 1} = \frac{4}{\sqrt{5} + 1}$

Now, we know that $\sqrt{7} > \sqrt{5}$ and $\sqrt{3} > 1$, add

So, $\sqrt{7} + \sqrt{3} > \sqrt{5} + 1$

$\Rightarrow \quad \frac{1}{\sqrt{7} + \sqrt{3}} < \frac{1}{\sqrt{5} + 1}$

$\Rightarrow \quad \frac{4}{\sqrt{7} + \sqrt{3}} < \frac{4}{\sqrt{5} + 1}$

$\Rightarrow \quad \sqrt{7} - \sqrt{3} < \sqrt{5} - 1$

So, $\sqrt{5} - 1 > \sqrt{7} - \sqrt{3}$

RATIONALIZATION OF SURDS

Rationalizing factor product of two surds is a rational number then each of them is called the rationalizing factor (R.F.) of the other. The process of converting a surd to a rational number by using an appropriate multiplier is known as **rationalization.**

Some examples :

(i) R.F. of $\sqrt{a}$ is $\sqrt{a}$ $\left(\therefore \sqrt{a} \times \sqrt{a} = a\right)$.

(ii) R.F. of $\sqrt[3]{a}$ is $\sqrt[3]{a^2}$ $\left(\therefore \sqrt[3]{a} \times \sqrt[3]{a^2} = \sqrt[3]{a^3} = a\right)$.

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(iii) R.F. of $\sqrt{a}+\sqrt{b}$ is $\sqrt{a}-\sqrt{b}$ & vice versa $\left[\because \left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=a-b\right]$.

(iv) R.F. of a $+\sqrt{b}$ is $a-\sqrt{b}$ & vice versa $\left[\because \left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=a^2-b\right]$

(v) R.F. of $\sqrt[3]{a}+\sqrt[3]{b}$ is $\left(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}\right)\left[\because \left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}\right)\right]$

$\left[\because \left(\sqrt[3]{a}\right)^3+\left(\sqrt[3]{b}\right)^3=a+b\right]$ which is rational.

(vi) R.F. of $\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right)is\left(\sqrt{a}+\sqrt{b}-\sqrt{c}\right)nd\left(a+b-c+2\sqrt{ab}\right)$.

Ex.9 Find the R.G. (rationalizing factor) of the following :

(i) $\sqrt{10}$ (ii) $\sqrt{12}$ (iii) $\sqrt{162}$ (iv) $\sqrt[3]{4}$ (v) $\sqrt[3]{16}$ (vi) $\sqrt[4]{162}$ (vii) $2+\sqrt{3}$

(viii) $7-4\sqrt{3}$ (ix) $3\sqrt{3}+2\sqrt{2}$ (x) $\sqrt[3]{3}+\sqrt[3]{2}$ (xi) $1+\sqrt{2}+\sqrt{3}$

(i) $\sqrt{10}$

Sol. $[\because \sqrt{10}\times\sqrt{10}=\sqrt{10\times10}=10]$ as 10 is rational number.

$\therefore$ R.F. of $\sqrt{10}$ is $\sqrt{10}$ **Ans.**

(ii). $\sqrt{12}$

Sol. First write it's simplest from i.e. $2\sqrt{3}$.

Now find R.F. (i.e. R.F. of $\sqrt{3}$ is $\sqrt{3}$)

$\therefore$ R.F. of $\sqrt{12}$ is $\sqrt{3}$ **Ans.**

(iii) $\sqrt{162}$

Sol. Simplest from of $\sqrt{162}$ is $9\sqrt{2}$.

R.F. of $\sqrt{2}$ is $\sqrt{2}$.

$\therefore$ R.F. of $\sqrt{162}$ is $\sqrt{2}$ **Ans.**

(iv) $\sqrt[3]{4}$

Sol. $\sqrt[3]{4}\times\sqrt[3]{4^2}=\sqrt[3]{4^3}=4$

$\therefore$ **R.F.** of $\sqrt[3]{4}$ is $\sqrt[3]{4^2}$ **Ans.**

(v). $\sqrt[3]{16}$

Sol. Simplest from of $\sqrt[3]{16}$ is $2\sqrt[3]{2}$

Now R.F. of $\sqrt[3]{2}$ is $\sqrt[3]{2^2}$

$\therefore$ R.F. of $\sqrt[3]{16}$ is $\sqrt[3]{2^2}$ **Ans.**

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(vi) $\sqrt[4]{162}$

Sol. Simplest form of $\sqrt[4]{162}$ is $3\sqrt[4]{2}$

Now R.F. of $\sqrt[4]{2}$ is $\sqrt[4]{2^3}$

R.F. of $(\sqrt[4]{162})$ is $\sqrt[4]{2^3}$ **Ans.**

(vii) $2+\sqrt{3}$

Sol. As $(2+\sqrt{3})(2-\sqrt{3}) = (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1,$, which is rational.

$\therefore$ R.F. of $(2+\sqrt{3})$ is $(2-\sqrt{3})$ **Ans.**

(viii) $7-4\sqrt{3}$

Sol. As $(7-4\sqrt{3})(7+4\sqrt{3}) = (7)^2 - (4-\sqrt{3})^2$ = 49 - 48 = 1, which is rational

$\therefore$ R.F. of $(7-4\sqrt{3})$ is $(7+4\sqrt{3})$ **Ans.**

(ix). $3\sqrt{3}+2\sqrt{2}$

Sol. As $(3\sqrt{3}+2\sqrt{2})(3\sqrt{3}-2\sqrt{2}) = (3\sqrt{3})^2 - (2\sqrt{2})^2 = 27 - 8 = 19$, which is rational.

$\therefore$ R.F. of $(3\sqrt{3}+2\sqrt{2})$ is $(3\sqrt{3}-2\sqrt{2})$ **Ans.**

(x) $\sqrt[3]{3}+\sqrt[3]{2}$

Sol. As $(\sqrt[3]{3}+\sqrt[3]{2})\left(\sqrt[3]{3^2} - \sqrt[3]{3}\times\sqrt[3]{2} + \sqrt[3]{2^2}\right) = \left(\sqrt[3]{3^3} + \sqrt[3]{2^3}\right)$ = 3 + 2 = 5, which is rational.

$\therefore$ R.F. of $(\sqrt[3]{3}+\sqrt[3]{2})$ is $\left(\sqrt[3]{3^2} - \sqrt[3]{3}\times\sqrt[3]{2} + \sqrt[3]{2^2}\right)$ **Ans.**

(xi) $1+\sqrt{2}+\sqrt{3}$

Sol. $(1+\sqrt{2}+\sqrt{3})(1+\sqrt{2}-\sqrt{3}) = (1+\sqrt{2})^2 - (\sqrt{3})^2$

$$= 1)^2 + (\sqrt{2})^2 + 2(1)(\sqrt{2}) - 3$$

$$= 1 + 2 + 2\sqrt{2} - 3$$

$$= 3 + 2\sqrt{2} - 3$$

$$= 2\sqrt{2}$$

$$2\sqrt{2}\times\sqrt{2} = 2\times 2 = 4$$

$\therefore$ R.F. of $1+\sqrt{2}+\sqrt{3}$ is $(1+\sqrt{2}-\sqrt{3})$ and $\sqrt{2}$. **Ans.**

NOTE : R.F. of $\sqrt{a}+\sqrt{b}$ or $\sqrt{a}-\sqrt{b}$ type surds are also called conjugate surds & vice versa.

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Ex.10 (i) $2-\sqrt{3}$ is conjugate of $2+\sqrt{3}$

(ii) $\sqrt{5}+1$ is conjugate of $\sqrt{5}-1$

NOTE : Sometimes conjugate surds and reciprocals are same.

Ex.11 (i) $2+\sqrt{3}$, it's conjugate is $2-\sqrt{3}$, its reciprocal is $2-\sqrt{3}$ & vice versa.

(ii) $5-2\sqrt{6}$, it's conjugate is $5+2\sqrt{6}$, its reciprocal is $5-2\sqrt{6}$ & vice versa.

(iii) $6-\sqrt{35}, 6+\sqrt{35}$

(iv) $7-4\sqrt{3}, 7+4\sqrt{3}$

(v) $8+3\sqrt{7}, 8-3\sqrt{7}$ .... and so on.

Ex.12 Express the following surd with a rational denominator.

Sol. $$\frac{8}{\sqrt{15}+1-\sqrt{5}-\sqrt{3}} = \frac{8}{\left[(\sqrt{15}+1)-(\sqrt{15}+\sqrt{3})\right]} \times \left[\frac{(\sqrt{15}+1)+(\sqrt{5}+\sqrt{3})}{(\sqrt{15}+1)+(\sqrt{5}+\sqrt{3})}\right]$$

$$= \frac{8(\sqrt{15}+1+\sqrt{5}+\sqrt{3})}{(\sqrt{15}+1)^2-(\sqrt{5}+\sqrt{3})^2}$$

$$= \frac{8(\sqrt{15}+1+\sqrt{5}+\sqrt{3})}{15+1+2\sqrt{15}-(5+3+2\sqrt{15})}$$

$$= \frac{8(\sqrt{15}+1+\sqrt{5}+\sqrt{3})}{8}$$

$$= (\sqrt{15}+1+\sqrt{5}+\sqrt{3})$$ **Ans.**

Ex.13 Rationalize the denominator of $\frac{a^2}{\sqrt{a^2+b^2}+b}$

Sol. $$\frac{a^2}{\sqrt{a^2+b^2}+b} = \frac{a^2}{\sqrt{a^2+b^2}+b} \times \frac{\sqrt{a^2+b^2}-b}{a^2+b^2-b}$$

$$= \frac{a^2\left(\sqrt{a^2+b^2}-b\right)}{\left(\sqrt{a^2+b^2}\right)^2-(b)^2}$$

$$= \frac{a^2\left(\sqrt{a^2+b^2}-b\right)}{a^2+b^2-b^2} = \left(\sqrt{a^2+b^2}-b\right)$$ **Ans.**

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Ex.14 If $\frac{3+2\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$, where a and b are rational then find the values of a and b.

Sol. L.H.S. $\frac{3+2\sqrt{2}}{3-\sqrt{2}} = \frac{(3+2\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$

$= \frac{9+3\sqrt{2}+6\sqrt{2}+4}{9-2}$

$= \frac{13+9\sqrt{2}}{7}$

$= \frac{13}{7} + \frac{9}{7}\sqrt{2}$

$\therefore \quad \frac{13}{7} + \frac{9}{7}\sqrt{2} = a + b\sqrt{2}$

Equating the rational and irrational parts

We get $a = \frac{13}{7}, b = \frac{9}{7}$ **Ans.**

Ex.15 If $\sqrt{3} = 1.732$, find the value of $\frac{1}{\sqrt{3}-1}$

Sol. $\frac{1}{\sqrt{3}-1} = \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{\sqrt{3}+1}{3-1}$

$= \frac{\sqrt{3}+1}{2}$

$= \frac{1.732+1}{2}$

$= \frac{2.732}{2}$

$= 1.366$ **Ans.**

Ex.16 If $\sqrt{5}$ = 2.236 and $\sqrt{2}$ = 1.414, then

Evaluate : $\frac{3}{\sqrt{5}+\sqrt{2}} + \frac{4}{\sqrt{5}-\sqrt{2}}$

Sol. $\frac{3}{\sqrt{5}+\sqrt{2}} + \frac{4}{\sqrt{5}-\sqrt{2}} = \frac{3\sqrt{5}-\sqrt{2}+4(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$

$= \frac{3\sqrt{5}-3\sqrt{2}+4\sqrt{5}+4\sqrt{2}}{5-2}$

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$= \frac{7\sqrt{5}+\sqrt{2}}{5-2}$

$= \frac{7\sqrt{5}+\sqrt{2}}{3}$

$= \frac{7\times 2.236+1.414}{3}$

$= \frac{15.652+1.414}{3}$

$= \frac{17.066}{3}$

= 5.689 (approximate)

Ex.17 If $= \frac{1}{2+\sqrt{3}}$ find the value of $x^3 - x^2 - 11x + 3$.

Sol. As, $x = \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$

$\Rightarrow$ $x - 2 = -\sqrt{3}$

$\Rightarrow$ $(x-2)^2 = \left(-\sqrt{3}\right)^2$ [By squaring both sides]

$\Rightarrow$ $x^2 + 4 - 4x = 3$

$\Rightarrow$ $x^2 - 4x + 1 = 0$

Now, $x^3 - x^2 - 11x + 3 = x^3 - 4x^2 + x + 3x^2 - 12x + 3$

$= x(x^2 - 4x + 1) + 3(x^2 - 4x + 1)$

$= x(0) + 3(0)$

$= 0 + 0 = 0$ **Ans.**

Ex.18 If $x = 3 - \sqrt{8}$, find the value of $x^3 + \frac{1}{x^3}$.

Sol. $x = 3 - \sqrt{8}$

$\therefore$ $\frac{1}{x} = \frac{1}{3-\sqrt{8}}$

$\Rightarrow$ $\frac{1}{x} = 3+\sqrt{8}$

Now, $x + \frac{1}{x} = 3-\sqrt{8}+3+\sqrt{8} = 6$

$\Rightarrow$ $x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3x\frac{1}{x}\left(x+\frac{1}{x}\right)$

$\Rightarrow$ $x^3 + \frac{1}{x^3} = (6)^3 - 3(6)$

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$\Rightarrow \quad x^3 + \frac{1}{x^3} = 216 - 18$

$\Rightarrow \quad x^3 + \frac{1}{x^3} = 198$ **Ans.**

Ex.19 If $x = 1 + 2^{1/3} + 2^{2/3}$, show that $x^3 - 3x^2 - 3x - 1 = 0$

Sol. $x = 1 + 2^{1/3} + 2^{2/3}$

$\Rightarrow \quad x - 1 (2^{1/3} + 2^{2/3})$

$\Rightarrow \quad (x - 1)^3 = (2^{1/3} + 2^{2/3})^3$

$\Rightarrow \quad (x - 1)^3 = (2^{1/3}) + (2^{2/3})^3 + 3.2^{1/3}.2^{2/3}(2^{1/3} + 2^{1/3-})$

$\Rightarrow \quad (x - 1)^3 = 2 + 2^2 + 3.2^1 (x - 1)$

$\Rightarrow \quad (x - 1)^3 = 6 + 6 (x - 1)$

$\Rightarrow \quad x^3 - 3x^2 + 3x - 1 = 6x$

$\Rightarrow \quad x^3 - 3x^2 - 3x - 1 = 0$ **Ans.**

Ex.20 Solve : $\sqrt{x+3} + \sqrt{x-2} = 5.$

Sol. $\sqrt{x+3} = 5 - \sqrt{x-2}$

$\Rightarrow \quad \left(\sqrt{x+3}\right)^2 = \left(5 - \sqrt{x-2}\right)^2$ [By squaring both sides]

$\Rightarrow \quad x + 3 = 25 + (x - 2) - 10\sqrt{x-2}$

$\Rightarrow \quad x + 3 = 25 + x - 2 - 10\sqrt{x-2}$

$\Rightarrow \quad 3 - 23 = - 10\sqrt{x-2}$

$\Rightarrow \quad -20 = -10\sqrt{x-2}$

$\Rightarrow \quad 2 = \sqrt{x-2}$

$\Rightarrow \quad x - 2 = 4$ [By squaring both sides]

$\Rightarrow \quad x = 6$ **Ans.**

Ex.21 If $x = 1 + \sqrt{2} + \sqrt{3}$, prove that $x^4 - 4x^3 - 4x^2 + 16 - 8 = 0.$

Hint : $x = 1 + \sqrt{2} + \sqrt{3}$

$\Rightarrow \quad x - 1 = \sqrt{2} + \sqrt{3}$

$\Rightarrow \quad (x-1)^2 = \left(\sqrt{2} + \sqrt{3}\right)^2$ [By squaring both sides]

$\Rightarrow \quad x^2 + 1 - 2x = 2 + 3 + 2\sqrt{6}$

$\Rightarrow \quad x^2 - 2x - 4 = 2\sqrt{6}$

$\Rightarrow \quad (x^2 - 2x - 4)^2 = (2\sqrt{6})^2$

$\Rightarrow \quad x^4 + 4x^2 + 16 - 4x^3 + 16x - 8x^2 = 24$

$\Rightarrow \quad x^4 - 4x^3 - 4x^2 + 16x + 16 - 24 = 0$

$\Rightarrow \quad x^4 - 4x^3 - 4x^2 + 16x - 8 = 0$ **Ans.**

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