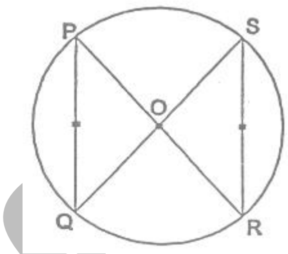
## 10.3 SOME IMPORTANT THEOREMS

**Theorem-1 : Equal chords of a circle subtend equal angles at the centre.**

**Given :** A circle with centre O in which chord PQ = chord RS.

**To Prove :** ∠POQ = ∠ROS.




**Proof :** In ΔPOQ and ΔROS,

OP = OR [Radii of the same circle]

OQ = OS [Radii of the same circle]

PW = RS [Given]

⇒ ΔPOQ = ΔROS [By SSS]

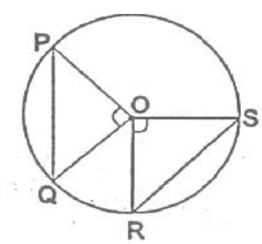
⇒ ∠POQ = ∠ROS [By cpctc] **Hence Proved.**

**Theorem-2 : If the angles subtended by the chords at the centre (of a circle) are equal then the chords are equal.**

**Given :** A circle with centre O . Chords PQ and RS subtend equal angles at the enter of the circle.

i.e. ∠POQ = ∠ROS

**To Prove :** Chord PQ = chord RS.




**Proof :** In ΔPOQ and ΔROS,

∠POQ = ∠ROS [Given]

OP = OR [Radii of the same circle]

OQ = OS [Radii of the same circle]

⇒ ΔPOQ ≅ ΔROS [By SSS]

⇒ chord PQ = chord RS [By cpctc] **Hence Proved.**

**Corollary-1 : Two arc of a circle are congruent, if the angles subtended by them at the centre are equal.**

**Corollary 2 : If two arcs of a circle are equal, they subtend equal angles at the centre.**

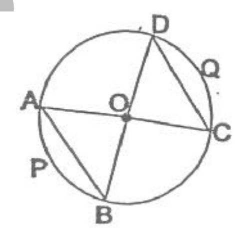
**Corollary 3 : If two arc of a circle are congruent (equal), their corresponding chords are equal.**

**Corollary 4: If two chords of a circle are equal, their corresponding arc are also equal.**

∠AOB = ∠COD

∴ Chord AB = Chord CD

∴ Arc APB = Arc COD.

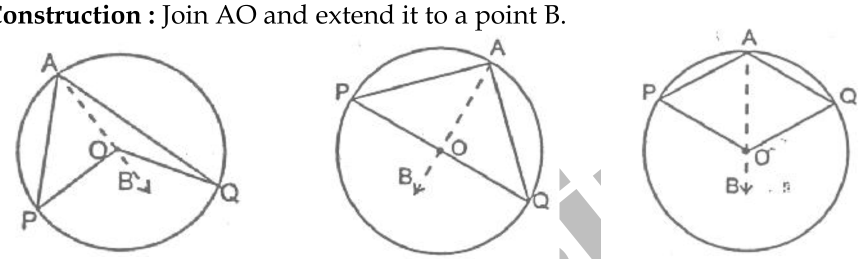


**Theorem-3 : The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.**

**Given :** An arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle.

**To Prove :** $\angle POQ = 2\angle PAQ$.

**Construction :** Join AO and extend it to a point B.




(A) (B) (C)

**Proof :** There arises three cases :

(A) are PQ is minor

(B) arc PQ s a semi - circle

(C) arc PQ is major.

In all the cases,

$\angle BOQ = \angle OAQ + \angle AQO$ ....(i)

[$\because$ An exterior angle of triangle is equal to the sum of the two interior opposite angles]

In OAQ,

OA = OQ [Radii of a circle]

$\therefore$ $\angle OAQ = \angle OQA$ ...(ii) [Angles opposite equal of a triangle are equal]

(i) and (ii), give,

$\angle BOQ = 2\angle OAQ$ ....(iii)

Similarly,

$\angle BOP = 2\angle OAP$ ....(iv)

Adding (iii) and (iv), we get

$\angle BOP + \angle BOQ = 2(\angle OAP + \angle OAQ)$

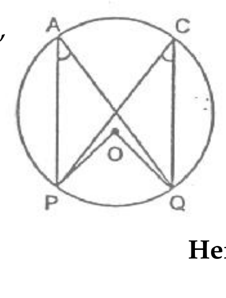
$\Rightarrow$ $\angle POQ = 2\angle PA$. ....(v)

**NOTE :** For the case (C), where PQ is the major arc, (v) is replaced by reflex angles.

Thus, $\angle POQ = 2\angle PAQ$.

**Theorem- 4 : Angles in the same segment of a circle are equal.**

**Proof :** Let P and Q be any two points on a circle to form a chord PQ, A and C any other points on the remaining part of the circle and O be the centre of the circle. Then,




$\angle POQ = 2\angle PAQ$ ...(i)

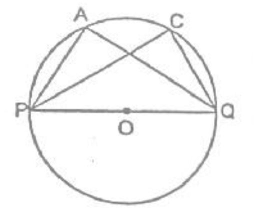
And $\angle POQ = 2\angle PCQ$ ...(ii)

From above equations, we get

$2\angle PAQ = 2\angle PCQ$

$\Rightarrow$ $\angle PAQ = \angle PCQ$ **Hence Proved**

**Theorem-5 : Angle in the semicircle is a right angle.**




**Proof :** $\angle PAQ$ is an angle in the segment, which is a semicircle.

$\therefore \quad \angle PAQ = \frac{1}{2}\angle PAO = \frac{1}{2} \times 180^0 = 90^0$

[$\therefore$ $\angle PQR$ is straight line angle or $\angle PQR = 180^0$]

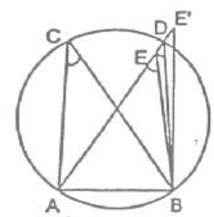
If we take any other point C on the semicircle, then again we get

$\angle PCQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 180^0 = 90^0$ **Hence Proved.**

**Theorem-6: If a line segment joining two points subtend equal angles at two other points lying on the same side of the lien containing the line segment the four points lie on a circle (i.e., they are concyclic).**

**Given :** AB is a line segment, which subtends equal angles at two points C and D. i.e., $\angle ACB = \angle ADB$.

**To Prove :** The points A, B, C and D lie on a circle.




**Proof :** Let us draw a circle through the points A, C and B.

Suppose it does not pass through the point D.

Then it will intersect AD (or extended AD) at a point, say E (or E′).

If points A,C,E and B lie on a circle,

$\angle ACD = \angle AEB$ [$\therefore$ Angles in the same segment of circle are equal]

But it is given that $\angle ACB = \angle ADB$

Therefore, $\angle AEB = \angle ADB$

This is possible only when E coincides with D. [As otherwise $\angle AEB > \angle ADB$]

Similarly, E′ should also coincide with D. So A, B, C and D are concyclic **Hence Proved.**