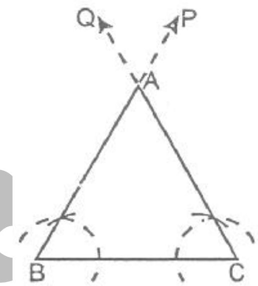
11.2 CONSTRUCTION OF A TRIANGLE

Case (i) To construct an equilateral triangle when its one side is given.

Ex.2 Draw an equilateral triangle having each side of 2.5 cm.

Sol. Given one side of the equilateral triangle be 2.5 cm.



STEPS :

(i) Draw a line segment BC = 2.5 cm.

(ii) Through B, construct ray BP making angle 60^0 with BC.

i.e. $\angle PBC = 60^0$

(iii) Through C, construct CQ making angle 60^0 with BC

i.e., $\angle QCB = 60^0$

(iv) Let BP and CQ intersect each other at point A.

The n, ΔABC is the require equilateral triangle.

Proof : Since, $\angle ABC = \angle ACB = 60^0$

$\therefore$ $\angle BAC = 180^0 - (60^0 + 60^0) = 60^0$

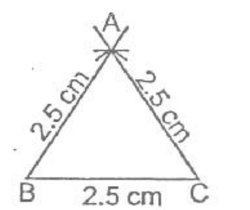
$\Rightarrow$ All the angles of the ΔABC drawn are equal.

$\Rightarrow$ All the sides of the ΔABC drawn are equal.

$\Rightarrow$ ΔABC is the required equilateral triangle. **Hence Proved.**

Alternate method :

If one side is 2.5 cm, then each side of the required equilateral triangle is 2.5 cm.



STEPS :

(i) Draw BC = 2.5 cm

(ii) With B as centre, draw an arc of radius 2.5 cm

(iii) With C as centre, draw an arc of radius 2.5 cm

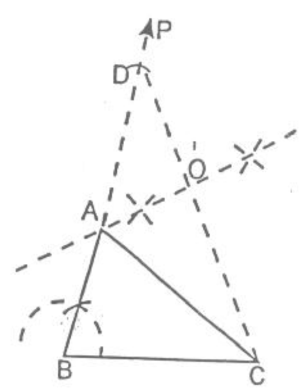
(iv) Let the two arc intersect each other at point A. Join AB and AC.

Then, ABC is the required equilateral triangle.

Case (ii) When the base of the triangle, one base angle and the sum of other two sides are given.

Ex.3 Construct a triangle with 3 cm base and sum of other two sides is 8 cm and one base angle is 60^0.

Sol. Given the base BC of the triangle ABC be 3 cm, one base angle $\angle B = 60^0$ and the sum of the other two sides be 8 cm i.e, AB + AC = 8 cm.



STEPS :

(i) Draw BC = 3 cm

(ii) At point B, draw PB so that $\angle PBC = 60^0$

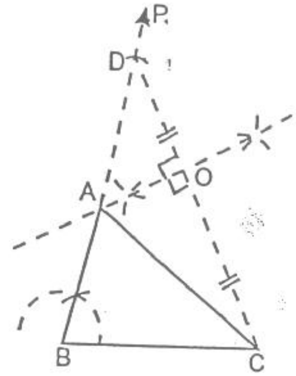
(iii) From BP, cut BC = 8 cm.

(iv) Join D and C.

(v) Draw perpendicular bisector of CD, which meets BD at point A.

(vi) Join A and C.

Thus, ABC is the required triangle.



Proof : Since, OA is perpendicular bisector of CD

$\Rightarrow$ OC = OD

$\angle AOC = \angle AOD = 90^0$

Also, OA = OA [Common]

$\therefore$ $\Delta AOC \cong \Delta AOD$ [By SAS]

$\Rightarrow$ AC = AD

$\therefore$ BD = BA + AD

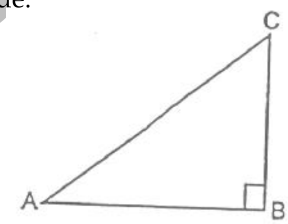
= BA + AC

= Given sum of the other two sides

Thus, base BC and $\angle B$ are draw as given and BD = AC. **Hence Proved.**

Ex.4 Construct a right triangle, when one side is 3.8 cm and the sum of the other side and hypotenuse is 6 cm.

Sol. Here, if we consider the required triangle to be ΔABC, as shown alongside.



Clearly, AB = 3.8 cm, $\angle B = 90^0$ and BC + AC = 6 cm.

STEPS :

(i) Draw AB = 3.8 cm

(ii) Through B, draw line BP so that $\angle ABP = 90^0$

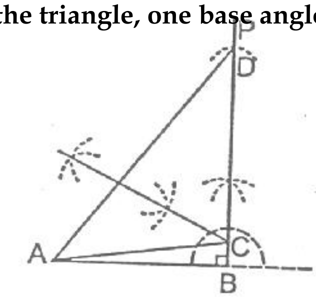
(iii) From BP, cut BD = 6 cm

(iv) Join A and D.

(v) Draw perpendicular bisector of AD, which meets BD at point C.

Thus, **ABC is the required triangle.**

Case (iii) When the base of the triangle, one base angle and the difference of the other two sides are given.

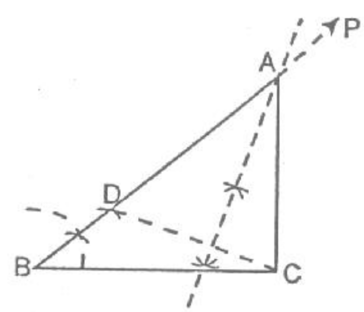


Ex.5 Construct a triangle with base of 8 cm and difference between the length of other two sides is 3 cm and one base angle is 60^0

Sol. Given the base BC of the required triangle ABC be 8 cm i.e., BC = 8 m, base angle B = 60^0 ant the difference between the lengths of other two sides AB and AC be 3 cm.

i.e., AB - AC = 3 cm or AC - AB = 3 cm.

(a) When AB - AC = 3 cm i.e., AB > AC :



STEPS :

(i) Draw BC = 8 cm

(ii) Through point B, draw BP so that $\angle PBC = 60^0$

(iii) From BP cut BD = 3 cm.

(iv) Join D and C.

(v) Draw perpendicular bisector of DC ; which meets BP at point A.

(vi) Join A and C.

Thus, ΔBC is the required triangle.

Proof : Since OA is perpendicular bisector of CD

$\Rightarrow$ OD = OC

$\angle AOD = \angle AOC = 90^0$

And, OA = OA [Common]

$\therefore$ $\Delta AOD \cong \Delta AOC$ [By SAS]

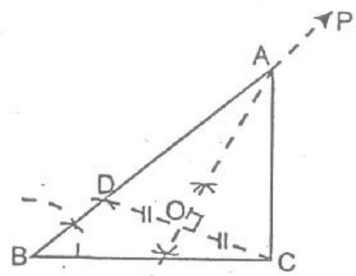
$\Rightarrow$ AD = AC [By cpctc]

Now, BD = BA - AD

= BA - AD

= BA - AC

= Given difference of other two sides.



Thus, the base BC and $\angle B$ are drawn as given and BD = BA - AC. **Hence Proved.**

(b) When AC - AB = 3 cm i.e, AB < AC :

STEPS :

(i) Draw BC = 8 cm

(ii) Through B, draw line BP so that angle PBC = 60^0.

(iii) Produce BP backward upto a suitable point Q.

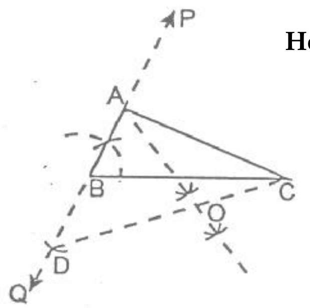
(iv) Fro BQ, cut BD = 3 cm.

(v) Join D and C.

(vi) Draw perpendicular bisector of DC, which meets BP at point A.

(vii) Join A and C.

Thus, ΔABC is the required triangle.



Proof : Since, OA is perpendicular bisector of CD

$\Rightarrow$ OD = OC

$\angle AOD = \angle AOC = 90^0$

And OA = OA [Common]

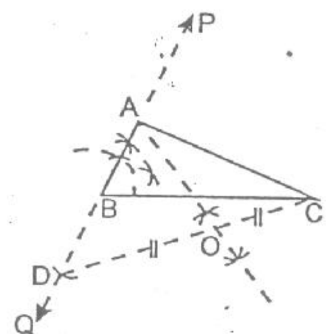
$\therefore$ $\Delta AOD \cong \Delta AOC$ [By SAS]

$\Rightarrow$ AD = AC [By cpctc]

Now, BD = AD - AB

= AC - AB [$\because$ AD = AC]

= Given difference of other two sides.



Thus, the base BC and $\angle B$ are drawn as given and BD = AC - AB. **Hence Proved.**

Case (iv) When the perimeter of the triangle and both the base angles are given :

Ex.6 Contruct a triangle ABC with AB + BC + CA = 12 cm $\angle B = 45^0$ and $\angle C = 60^0$

Sol. Given the perimeter of the triangle ABC be 12 cm i.e., AB + BC + CA = 12 cm and both the base angles be 45^0 and 60^0 i.e., $\angle B = 45^0$ and $\angle C = 60^0$

STEPS :

(i) Draw a line segment PQ = 12 cm

(ii) At P, construct line PR so that $\angle RPO = 45^0$ and at Q, construct a line QS so that $\angle SQP = 60^0$

(iii) Draw bisector of angles RPQ and SQP which meet each other at point A.

(iv) Draw perpendicular bisector of AP, which meets PQ at point B.

(v) Draw perpendicular bisector of AQ, which meets PQ at point C.

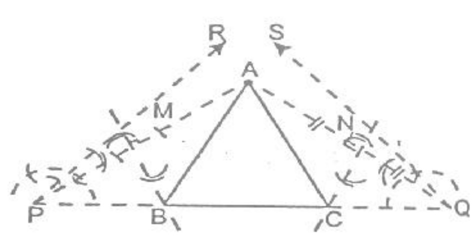
(vi) Join AB and AC.

Thus, ABC is the required triangle.

Proof : Since, MB is perpendicular bisector of AP

$\Rightarrow$ $\Delta QNC \cong \Delta ANC$ [By SAS]

PB = AC



Similarly, NC is perpendicular bisector of AQ.

$\Rightarrow$ $\Delta QNC \cong \Delta ANC$ [By SAS]

$\Rightarrow$ CQ = AC [By cpctc]

Now, PQ = PB + BC + CQ

= AB + BC + AC

= Given perimeter of the ΔABC drawn.

Also, $\angle BPA = \angle BAP$ [As Δ PMB $\cong \Delta$ A MB]

$\therefore$ $\angle ABC = \angle BPA + \angle BAP$ [Ext. angle of a triangle = sum of two interior opposite angles]

$\angle ABC = \angle BPA + \angle BAP = 2\angle BPA = \angle RPB = \angle ACB$ [Given]

$\angle ACB = \angle CQA + \angle CQA$

$= 2\angle CQA$ [$\because \Delta QNC \cong \Delta ANC \therefore \angle CQA = \angle CAQ$]

$= \angle SQC$ = Given base angle ACB.

Thus, given perimeter = perimeter of ΔABC.

given one base angle = angle ABC

and, given other base angle = angle ACB.

Ex.7 Construct and equilateral triangle if its altitude is 3.2 cm.

Sol. Given In an equilateral ΔABC an altitude AD = 3.2 cm

Required to Construct an equilateral triangle ABC from the given data.

STEPS :

(i) Draw a line PQ and mark and point D on it.

(ii) Construct a ray DE perpendicular to PQ.

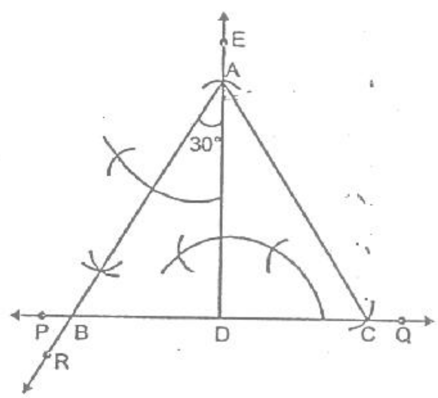
(iii) Cut off DA = 3.2 cm from DE.

(iv) Construct $\angle DAR = 30^0 = \left(\frac{1}{2} \times 60^0\right)$.

The ray AR intersects PQ at B.

(v) Cut off line segment DC = BD.

(vi) Join A and C. We get the required ΔABC.



Ex.7 Construct a right angled triangle whose hypotenuse measures 8 cm and one side is 6 cm.

Sol. **Given Hypotenuse AC of a** ΔABC = 8 cm and one side AB = 6 cm.

Required To construct a right angled ΔABC from the given data.

STEP.

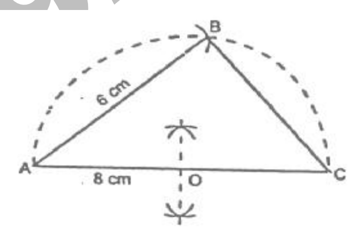
(i) Draw a line segment AC = 8 cm.

(ii) Mark the mid point O of AC.

(iii) With O as centre and radius OA, draw a semicircle on AC.

(iv) With A as centre and radius equal to 6 cm, draw an arc, cutting the semicircle a B.

(v) Jon A and B ; B and C. We get the required right angled triangle ABC.

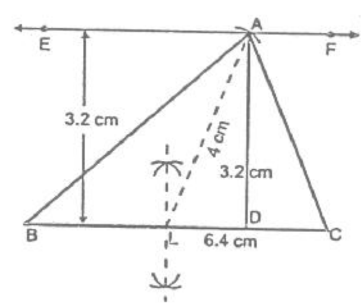


Ex.8 Construct a ΔABC in which BC = 6.4 cm, altitude from A is 3.2 cm and the median bisecting BC is 4 cm.

Sol. **Given :** one side BC = 6.4 cm, Altitude AD = 3.2 am and the median AL = 4 cm.

Required : To construct a ΔABC form the given data.

STEP :



(i) Draw BC = 6.4 cm

(ii) Bisect BC at L.

(iii) Draw EF ∥ BC at a distance 3.2 cm for BC.

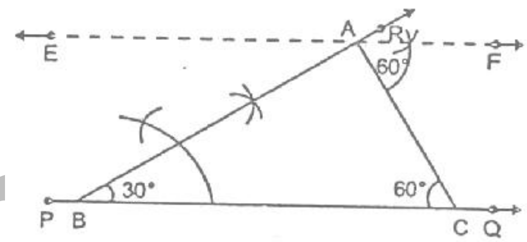
(iv) With L as centre and radius equal to 4 cm, draw an arc, cutting EF at A.

(v) Join A and B ; A and C. We get the required triangle ABC.

Ex.9 Construct a ΔABC in which ∠B = 30^0 and ∠C = 60^0 and the perpendicular from the vertex A to the base BC is 4.8 cm.

Sol. **Given :** ∠B = 30^0 ∠C = 60^0, length of perpendicular from vertex A to be base BC = 4.8 cm.

Required : To construct a ΔABC from the given data.



STEP :

(i) Draw any ray line PQ.

(ii) Take a point B on line PQ and construct ∠QBR = 30^0

(iii) Draw a line EF ∥ PQ a distance of 4.8 cm from PQ, cutting BR at A.

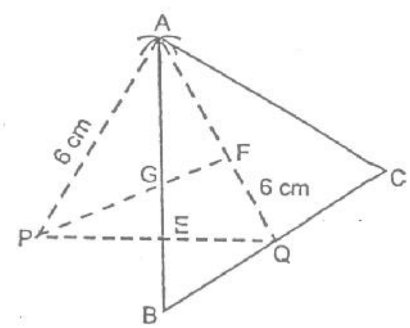
(iv) Construct an angle ∠FAC = 60^0, cutting PQ C.

(v) Join A and C. We get the required triangle ABC.

Ex.10 Construct a triangle ABC, the lengths of whose medians are 6 cm, 7cm and 6 cm.

Sol. **Given :** Median AD = 6 cm Median BE = 7 cm Median CF = 6 cm.

Required : To construct a ΔABC from the given data.



STEP :

(i) Construct a ΔAPQ with AP = 6 cm, PQ = 7 cm and AQ = 6 cm.

(ii) Draw the medians AE and PF of ΔAPQ intersecting each other at G.

(iii) Produce AE to B such that GE = EB

(iv) Join B and Q and produce it to C, such that BQ = QC

(v) Join A and C. We get the required triangle ABC.