CHAPTER – 13
SURFACE AREA AND VOLUMES

13.1 INTRODUCTION AND FORMULAE OF MENSURATION

If any figure such as cuboids, which has three dimensions length, width and height are known as three dimensional figures. Where are rectangle has only two dimensions i.e. length and width. Three dimensional figures have volume in addition to areas of surface from which these solid figures are formed.

(a) Cuboids :

There are six faces (rectangular), eight vertices and twelve edges in a cuboids.

Total Surface Area (T.S.A.) : The area of surface from which cuboids is formed.

(i) Total Surface Area (T.S.A.) $= 2[\ell \times b + b \times h + h \times \ell]$

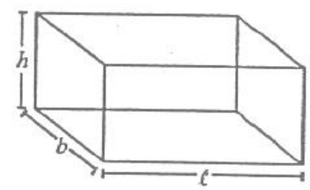
(ii) Lateral Surface Area (L.S.A.) $= 2[b \times h + h \times \ell]$

(or Area of 4 walls) $= 2h[\ell + b]$

(iii) Volume of Cuboids $=$ (Area of base) $\times$ height

$= (\ell \times b) \times h$

(iv) Length of diagonal $= \sqrt{\ell^2 + b^2 + h^2}$



(b) Cube :

Cube ahs six faces. Each face is a square.

(i) T.S.A. $= 2[x.x + x.x + x.x]$

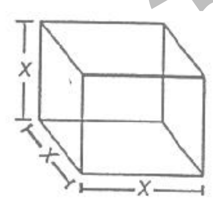
$= 2[x^2 + x^2 + x^2] = 2(33^2) = 6x^2$

(ii) L.S.A. $= 2[x^2 + x^2] = 4x^2$

(iii) Volume $=$ (Area of base) $\times$ Height

$= (x^2) \times x = x^3$

(iv) Length of diagonal $= x\sqrt{3}$



(c) Cylinder :

Curved surface area of cylinder (C.S.A.) : It is the area of surface from which the cylinder is formed. When we cut this cylinder, we will find a rectangle with length $2\pi r$ are height h units.

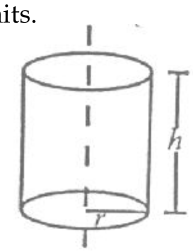
(i) C.S.A. of cylinder $= (2\pi r) \times h = 2\pi rh.$

(ii) T.S.A $=$ C.S.A. + circular top & bottom

$= 2\pi rh + (\pi r^2) + (\pi r^2)$

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$ sq. units



(iii) Volume of cylinder = Area of base × height

$= (\pi r^2) \times h$

$= \pi r^2 h$ cubic units

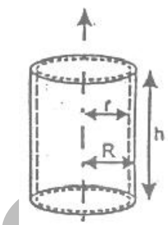
Hollow cylinder :

(i) C.S.A. of hollow cylinder = $2\pi (R + r)h$ sq. units

(ii) T.S.A. of hollow cylinder = $2\pi (R + r)h + \pi (R^2 - r^2)$

$= \pi (R + r) [2h + R - r]$ sq. units

(iii) Volume of hollow cylinder = $\pi (R^2 - r^2)h$ cubic units



Where, r = inner radius of cylinder

R = outer radius of cylinder

h = height of the cylinder

(d) Cone :

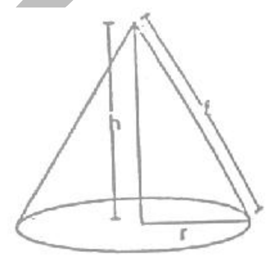
(i) C.S.A. of cone $= \pi\ell$

(ii) T.S.A . of cone = C.S.A. + Base area

$= \pi r\ell + \pi r^2$

$= \pi r(\ell + r)$

(iii) Volume of cone $= \frac{1}{3}\pi r^2 h$



Where, h = height

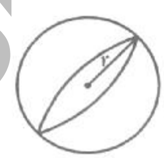
r = radius of base

ℓ = slant height

(e) Sphere :

(i) T.S.A. of sphere $= 4\pi r^2$

(ii) Volume of sphere $= \frac{4}{3}\pi r^3$



(f) Hemisphere :

(i) C.S.A. $= 2\pi r^2$

(ii) T.S.A. = C.S.A. + other area

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

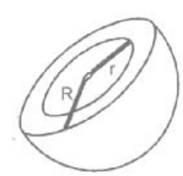
(iii) Volume $= \frac{2}{3}\pi r^3$

(g) Hollow Hemisphere :

(i) C.S.A. $= 2\pi (R^2 + r^2)$

(ii) T.S.A $= 2\pi (R^2 + r^2) + \pi (R^2 - r^2)$

(iii) Volume $= \frac{2}{3}\pi (R^3 - r^3)$



ILLUSTRATIONS :

Ex.1 Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the new cuboids to that of the sum of the surface areas of three cubes.

Sol. Let the side of each of the three equal cubes be a cm.

Then surface area of one cube = $6a^2$ cm^2

$\therefore$ Sum of the surface areas of three cubes = $3 \times 6a^2 = 18a^2$ cm^2

For new cuboids

length (ℓ) = 3a cm

breadth (b) = a cm

height (h) = a cm

$\therefore$ Total surface are of the new cuboids $= 2(\ell \times b + b \times h + h = \ell)$

$= 2[3a \times a + a \times a + a \times 3a]$

$= 2[3a^2 + a^2 + 3a^2] = 14a^2$ cm.2

$\therefore$ Required ratio $= \dfrac{\text{Total surface area of the new cuboid}}{\text{Sum of the surace areas of three cubes}}$

$= \dfrac{14a^2}{18a^2} = \dfrac{7}{9} = 7 : 9$ **Ans.**

Ex.2 A class room is 7 m long, 6.5 m wide and 4 m high. It has one door 3 m × 1.4 m and three windows each measuring 2 m × 1 m The interior walls are to be colour-washed. The contractor charges Rs. 15 per sq. m. Find the cost of colour washing.

Sol. ℓ = 7m, n = 6.5 m and h = 4 m

$\therefore$ Area of the room $= 2(\ell + b)h = 2(7 + 6.5)\,4 = 108$ m^2

Area of door $= 3 \times 1.4 = 4.2$ m^2

Area of one window $= 3 \times 2 = 6$ m^2

$\therefore$ Area of 3 windows $= 3 \times 2 = 6$ m^2

$\therefore$ Area of the walls of the room to be colour washed $= 108 - (4.2 + 6)$

$= 108 - 10.2 = 97.8$ m^2

$\therefore$ Cost of colour washing @ Rs. 15 per square metre = Rs. 97.8 × 15 = Rs. 1467. **Ans.**

Ex.3 A cylindrical vessel, without lid, has to be tin coated including both of its sides. If the radius of its base is $\dfrac{1}{2}$ m and its height is 1.4 m, calculate the cost of tin-coating at the rate of Rs. 50 per 1000 cm^2.

Sol. Radius of the base (r) $= \dfrac{1}{2}$ m

$= \dfrac{1}{2} \times 100$ cm = 50 cm

Height (h) = 1.4 m

= 1.4 × 100 cm

= 140 cm.

Surface area of to tin-coated $= 2\,(2\pi r + \pi r^2)$

$= 2[2 \times 3.14 \times 50 \times 140 + 3.14 \times (50)^2]$

$= 2[43960 + 7850] = 2(51810) = 103620$ cm^2

$\therefore$ Cost of tin-coating at the rate or Rs. 50 per 1000 cm^2

$= \dfrac{50}{1000} \times 103620$ = Rs 5181. **Ans.**

Ex.4 The diameter of a roller 120 cm long is 84 cm. If its takes 500 complete revolutions to level a playground determine the cost of leveling at the rate of Rs. 25 per square metre. (Use $\pi = \frac{22}{7}$)

Sol. $2r = 84$ cm

$\therefore$ $r = \frac{84}{2}$ cm = 42 cm

$h = 120$ cm

Area of the playground leveled in one complete revolution = $2\pi rh$

$= 2 \times \frac{22}{7} \times 42 \times 120 \times 31680 \text{ cm}^2$

$\therefore$ Area of the playground = $31680 \times 500 \text{ cm}^2$

$= \frac{31680 \times 500}{100 \times 100} \text{ m}^2 = 1584 \text{ m}^2$

$\therefore$ Cost of leveling @ Rs 25 per square metre = Rs 1584 × 25 = 39600. **Ans.**

Ex.5 How many metres of cloth of 1.1 m width will be required to make a conical tent whose vertical height is 12 m and base radius is 16 m ? Find also the cost of the cloth used at the rate of Rs 14 per metre.

Sol. $h = 12$ m

$r = 16$ m

$\therefore$ $\ell = \sqrt{r^2 + h^2}$

$= \sqrt{(16)^2 + (12)^2} = \sqrt{256 + 144}$

$= \sqrt{400} = 20\text{m}$

$\therefore$ Curved surface area = $\pi r\ell = \frac{22}{7} \times 16 \times 20 = \frac{7040}{7}\text{m}^2$

Width of cloth = 1.1 m

$\therefore$ Length of cloth = $\frac{7040/7}{1.1} = \frac{70400}{77} = \frac{6400}{7}\text{m}$

$\therefore$ Cost of the cloth used @ Rs 14 per metre = Rs $\frac{6400}{7} \times 14$ = Rs 12800 **Ans.**

Ex.6 The surface area of a sphere of radius 5 cm is five times the area of the curved surface of cone of radius 4 cm. Find the height of the cone.

Sol. Surface area of sphere of radius 4 cm = $\pi (4) \ \ell \text{ cm}^2$ when ℓ cm is the slant height of the cone.

According to the question,

$4\pi(5)^2 = 5[\pi(4)\ell]$

$\Rightarrow$ $\ell = 5$ cm $\Rightarrow$ $\sqrt{r^2 + h^2} = 5$

$\Rightarrow$ $r^2 + h^2 = 25$ $\Rightarrow$ $(4)^2 + h^2 = 25$

$\Rightarrow$ $16 + h^2 = 25$ $\Rightarrow$ $h^2 = 9$

$\Rightarrow$ $h = 3$

Hence the height of the cone is 3 cm. **Ans.**

Ex.7 The dimensions of a cinema hall are 100 m, 50 m and 18m. How many persons can sit in the hall, if each required 150 m^3 of air ?

Sol. $\ell = 100$ m

b = 50 m

h = 18 m

$\therefore$ Volume of the cinema hall = ℓbh

$= 100 \times 50 \times 18 = 90000 \text{ m}^3$

Volume occupied by 1 person = 150 m^3

$\therefore$ Number of persons who can sit in the hall = $\frac{\text{Volume of the ball}}{\text{Volume occupied by 1 person}}$

$$= \frac{90000}{150} = 600$$

Hence 600 persons can sit in the hall. **Ans.**

Ex.8 The outer measurements of a closed wooden box are 42 cm, 30 m and 27 cm. If the box is made of 1 cm thick wood, determine the capacity of the box.

Sol. **Outer dimensions**

ℓ = 42 cm

b = 30 cm

h = 27 cm

Thickness of wood = 1 cm

Inner dimensions

$\ell = 42 - (1 + 1) = 40$ cm

$b = 30 - 1(1 + 1) = 28$ cm

$h = 27 - (1 + 1) = 25$ cm

$\therefore$ Capacity of the box $\ell \times b \times h$

$= 40 \times 28 \times 25 = 28000 \text{ cm}^2$. **Ans.**

Ex.9 If v is the volume of a cuboids of dimensions a,b, and c and s is its surface area, then prove that $\frac{1}{v} = \frac{2}{a}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

Sol. L.H.S. = $\frac{1}{v} = \frac{1}{abc}$ ....(i)

R.H.S. = $\frac{2}{s}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

$= \frac{2}{2(ab + bc + ca)}\left(\frac{bc + ca + ab}{abc}\right)$

$= \frac{1}{abc}$ ....(ii)

from (i) and (ii) $\frac{1}{v} = \frac{2}{s}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$. **Hence Proved.**

Ex.10 The ratio of the volumes of the two cones is 4 : 5 and the ratio of the radii of their bases is 2 : 3. Find the ratio of their vertical heights.

Sol. Let the radii of bases, vertically heights and volumes of the two cones be r_1, h_1, v_1 and r_2, h_2, v_2 respectively.

According to the question,

$$\frac{v_1}{v_2} = \frac{4}{5} \qquad ...(i) \qquad\qquad \frac{r_1}{r} = \frac{2}{3} \qquad ....(i)$$

From (i), we have $\dfrac{\frac{1}{2}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \dfrac{4}{5}$

$$\Rightarrow \frac{r_1^2 h_1}{r_2^2 h_2} = \frac{4}{5}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 \frac{h_1}{h_2} = \frac{4}{5}$$

$$\Rightarrow \left(\frac{2}{3}\right)^2 = \frac{h_1}{h_2} = \frac{4}{5}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{4}{5}\left(\frac{3}{2}\right)^2 \quad \text{[Using (ii)]}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{9}{5}$$

Hence the ratio of their vertical height is 9 : 5. **Ans.**

Ex.11 If h, c and v be the height, curved surface and volume of a cone, show that $3\pi vh^3 - c^2h^2 + 9v^2 = 0$.

Sol. Let the radius of the base and slant height of the cone be r and ℓ respectively. The n ;

$$c = \text{curved surface} = \pi r\ell = \pi r\sqrt{r^2 + h^2} \qquad ....(i)$$

$$v = \text{volume} = \frac{1}{3}\pi r^2 h \qquad ...(ii)$$

$$\therefore \quad 3\pi vh^3 - c^2h^2 + 9v^2 = 3\pi(\frac{1}{3}\pi r^2 h)h^2 - \pi^2 r^2 (r^2 + h^2)h^2 + 9\left(\frac{1}{3}\pi r^2 h\right)^2 \quad \text{[Using (i) and (ii)]}$$

$$=^2 r^2h^4 - \pi^2r^2h^2 - \pi^2r^2h^4 + \pi^2r^4h^2 = 0.$$ **Hence Proved.**

Ex.12 How many balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm ?

Sol. Volume of the spherical ball of radius 8 cm = $\frac{4}{3}\pi \times 8^3 c^3$

Also, volume of each smaller spherical ball of radius 1 cm $= \frac{4}{3\pi \times 1^3} cm^3$.

Let n be the number of smaller balls that can be made. Then, the volume of the larger ball is equal to the sum of all the volumes of n smaller balls.

Hence, $\frac{4}{3}\pi \times n = \frac{4}{3}\pi \times 8^3$

$\Rightarrow \quad n = 8^3 = 512$

Hence, the required number of balls = 512. **Ans.**

Ex.13 By melting a solid cylindrical metal, a few conical materials are to be made. If three times the radius of the cone is equal to twice the radius of the cylinder an the ratio of the height of the cylinder and the height of the cone is 4 : 3, find the number of cones which can be made.

Sol. Let R be the radius and H be the height of the cylinder and let r and h be the radius and height of the cone respectively. Then,

$3r = 2R$

And $H ; h = 4 : 3$ ....(i)

$\Rightarrow \frac{H}{h} = \frac{4}{3}$

$\Rightarrow 3H = 4h$ ....(ii)

Let n be the required number of cones which can be made from the materials of the cylinder. Then, the volume of the cylinder will be equal to the sum of the volumes of n cones. Hence, we have

$$\pi R^2 H = \frac{n}{3}\pi r^2 h$$

$\Rightarrow 3R^2H = nr^2h$

$$\Rightarrow n = \frac{3R^2H}{r^2h} = \frac{3 \times \frac{9r^2}{4} \times \frac{4h}{3}}{r^2h}$$ [$\because$ From (i) and (ii), $R = \frac{3r}{2}$ and $H = \frac{4h}{3}$]

$$= \frac{3 \times 9 \times 4}{3 \times 4} = 9$$

Hence, the required number of cones is 9. **Ans.**

Ex.14 Water flows at the rate of 10 per minute through a cylindrical pipe having its diameter as 5 mm. How much time will it take to fill a conical vessel whose diameter of the base is 40 cm and depth 24 cm ?

Sol. Diameter of the pipe = 5 mm = $\frac{5}{10}$cm = $\frac{1}{2}$cm.

$\therefore$ Radius of the pipe = $\frac{1}{2} \times \frac{1}{2}$cm = $\frac{1}{4}$ cm.

In 1 minute, the length of the water column in the cylindrical pipe = 10 m = 1000 cm.

$\therefore$ Volume of water that flows out of the pipe in 1 minute = $\pi \times \frac{1}{4} \times \frac{1}{4} \times 1000$ cm^3.

Also, volume of the cone = $\times \frac{1}{3} \times \pi \times 20 \times 20 \times 24$ cm^3.

Hence, the time needed to fill up this conical vessel $= \left(\frac{1}{3}\pi \times 20 \times 20 \times 24 \div \pi \times \frac{1}{4} \times \frac{1}{4} \times 1000\right)$ minutes

$$= \left(\frac{20 \times 20 \times 24}{3} \times \frac{4 \times 4}{100}\right) = \frac{4 \times 24 \times 16}{30} \text{ minutes} = \frac{256}{5} \text{ minutes} = 51.2 \text{ minutes.}$$

Hence, the required time is 51.2 minutes. **Ans.**