2.4 ZEROS OR ROOTS OF A POLYNOMIAL

A real number α is a root or zero of polynomial f(x) = $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0$,

if $f(\alpha) = 0$. i.e. $a_n \alpha^n + a_{n-1} + a_{n-2}\alpha^{n-2} + ... + a_1\alpha + a_0 = 0$.

For example x = 3 is root of the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$, because

$f(3) = (3)^3 - 6(3)^2 + 11(3) - = 27 - 54 + 33 - 6 = 0$.

but x = - is not a root of the above polynomial,

$\because \quad f(-2) = (-2).3 - 6(-2)^2 + 11(-2) - 6$

$f(-2) = -8 - 24 - 22 - 6$

$f(-2) = -60 \neq 0$.

(a) Value of a Polynomial :

The value of a polynomial f(x) at x= α is obtained by substituting x = α in the given polynomial and is denoted by $f(\alpha)$. e.g. If $f(x) = 2x^3 - 13x^2 + 17x + 12$ then its value at x = 1 is.

$f(1) = 2(1)^3 - 13(1)^2 + 17(1) + 12$

$= 2 - 13 + 17 + 12 = 18$.

Ex.1 Show that x = 2 is a root of $2x^3 + x^2 - 7x - 6$.

Sol. $p(x) = 2x^3 + x^2 - 7x - 6$ then,

$p(2) = 2(2)^3 + (2)^2 7(2) - 6 = 16 + 4 - 14 - 6 = 0$

Hence x = 2 is a root of p(x). **Ans.**

Ex. 2 If $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.

Sol. $f(x) = 6x^3 - 11x^2 + kx - 20$

$\Rightarrow \quad f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$

$\Rightarrow \quad 6.\frac{64}{9.3} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$

$\Rightarrow \quad 128 - 176 + 12k - 180 = 0$

$\Rightarrow \quad 12k + 128 - 356 = 0$

$\Rightarrow \quad 12k = 228$

$\Rightarrow \quad k = 19$ **Ans.**

Ex.3 If $x = 2$ & $x = 0$ are two roots of the polynomial $f(x) = 2x^3 - 5x^2 + ax + b$. Find the values of and b.

Sol. $f(x) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow$ $16 - 20 + 2a + b = 0$

$\Rightarrow$ $2a + b = 4$ ....(i)

$\Rightarrow$ $f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0$

$\Rightarrow$ $b = 0$

So, $2a = 4$

Hence, $a = 2$, $b = 0$ **Ans.**