2.6 FACTORISATION OFA QUADRATIC POLYNOMIAL

For factorisation of a quadratic expression $ax^2 + bx + a$ where $a \neq 0$, there are two method.

(a) By Method of Completion of Square :

In the form $ax^2 + bx + c$ where $a \neq 0$, firstly we take 'a' common in the whole expression then factorise by converting the expression $a\left\{x^2 + \frac{b}{a}x + \frac{c}{a}\right\}$ as the difference of two squares.

Ex.10 Factorise $x^2 - 31x + 220$.

Sol. $x^2 - 31a + 220$

$$= x^2 - 2.\frac{31}{2}.x + \left(\frac{31}{2}\right)^2 - \left(\frac{31}{2}\right)^2 + 220$$

$$= \left(x - \frac{31}{2}\right)^2 - \frac{961}{4} + 220 = \left(x - \frac{31}{2}\right)^2 - \frac{81}{4}$$

$$= \left(x - \frac{31}{2}\right) - \left(\frac{9}{2}\right)^2 = \left(x - \frac{31}{2} + \frac{9}{2}\right)\left(x - \frac{31}{2} - \frac{9}{2}\right)$$

$= (x - 11)(x - 20)$ **Ans.**

Ex.11 Factorise : $-10x^2 + 31x - 24$

Sol. $-10x^2 + 31x - 24$

$$= -[10x^2 - 31x + 24] = -10\left[x^2 - \frac{31}{10}x + \frac{24}{10}\right]$$

$$= -10\left[x^2 - 2.\frac{31}{20}.x + \left(\frac{31}{20}\right)^2 - \left(\frac{31}{20}\right)^2 + \frac{24}{10}\right]$$

$$= -10\left[\left(x - \frac{31}{20}\right)^2 - \frac{961}{400} + \frac{24}{10}\right] = -10\left[\left(x - \frac{31}{20}\right)^2 - \frac{1}{400}\right]$$

$$= -10\left[\left(x - \frac{31}{20}\right)^2 - \left(\frac{1}{20}\right)^2\right] = -10\left[x - \frac{31}{20} + \frac{1}{20}\right]\left[x - \frac{31}{20} - \frac{1}{20}\right]$$

$$= -10\left(\frac{2x-3}{2}\right)\left(\frac{5x-8}{5}\right) = -(2x-3)(5x-8) = (3-2x)(5x-8)$$ **Ans.**

(b) By Splitting the Middle Term :

In the quadratic expression $ax^2 + bx + c$, where a is the coefficient of x^2, b is the coefficient of x and c is the constant term. In the quadratic expression of the form $x^2 + bx + c$, a = 1 is the multiple of x^2 and another terms are the same as above.

There are four types of quadratic expression :

(i) $ax^2 + bx + c$ (ii) $ax^2 - bx + c$

(iii) $ax^2 - bx - c$ (iv) $ax^2 + bx - c$

Ex.12 Factorise : $2x^2 + 12\sqrt{2x} + 35$.

Sol. $2x^2 + 12\sqrt{2x} + 35$

Product ac = 70 & b = $12\sqrt{2}$

$\therefore$ Split the middle term as $7\sqrt{2}\,\&\,5\sqrt{2}$

$\Rightarrow$ $2x^2 + 12\sqrt{2}x + 35 = 2x^2 + 7\sqrt{2}x + 5\sqrt{2}x + 35$

$= \sqrt{2}x\left[\sqrt{2}x+7\right]+5\left[\sqrt{2}x+7\right]$

$= \left[\sqrt{2}x+5\right]\left[\sqrt{2}x+7\right]$ **Ans.**

Ex.13 Factorise : $x^2 - 14x + 24$.

Sol. Product ac = 24 & b = -14

$\therefore$ Split the middle term as - 12 & - 2

$\Rightarrow$ $x^2 - 14x + 24 = x^2 - 12 - 2x + 24$

$\Rightarrow$ $x(x - 12) - 2(x - 12) = (x - 12)(x - 2)$ **Ans.**

Ex.14 Factorise : $x^2 - \frac{13}{24}x - \frac{1}{12}$.

Sol. $x^2 - \frac{13}{24}x - \frac{1}{12} = \frac{1}{24}[24x^2 - 13x - 2]$

Product ac = - 48 & b = - 13 $\therefore$ We split the middle term as - 16x + 3x.

$= \frac{1}{24}[24x^2 - 16x + 3x - 2]$

$= \frac{1}{24}[8x(3x - 2) + 1(3x - 2)]$

$= \frac{1}{24}(3x - 2)(8x + 1)$ **Ans.**

Ex.15 Factorise : $\frac{3}{2}x^2 - 8x - \frac{35}{2}$

Sol. $\frac{3}{2}x^2 - 8x - \frac{35}{2} = \frac{1}{2}(3x^2 - 16x - 35) = \frac{1}{2}(3x^2 - 21x + 5x - 35)$

$= \frac{1}{2}[3x(x-7) + 5x(x-7)] = \frac{1}{2}(x-7)(3x+5)$ **Ans.**

(c) Integral Root Theorem :

If f(x) is a polynomial with integral coefficient and the leading coefficient is 1, then any integer root of f(x) is a factor of the constant term. Thus if $f(x) = x^3 - 6x^2 + 11x - 6$ has an Integral root, then it is one of the factors of 6 which are $\pm 1, \pm 2, \pm 3, \pm 6$. .

Now Infect $f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

$f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$

$f(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$

Therefore Integral roots of f(X) are 1,2,3.

NOTE:

(i) An n^{th} degree polynomial can have at most n real roots.

(ii) Finding a zero or root of polynomial f(x) means solving the polynomial equation f(x) = 0. It follows from the above discussion that if $f(x) = ax + b,\ a \neq 0$ is a linear polynomial, then it has only one root given by f(x) = 0 i.e. f(x) = ax + b = 0

$\Rightarrow \quad ax = -b$

$\Rightarrow \quad x = -\frac{b}{a}$

Thus $a = -\frac{b}{a}$ is the only root of f(x) = ax + b.

Ex.16 If $f(x) = 2x^3 - 13x^2 + 17x + 12$ then find out the value of f(-2) & f(3).

Sol. $f(x) = 2x^3 - 13x^2 + 17x + 12$

$f(-2) = 2(-2)^3 - 13(-2)^2 + 17(-2) + 12$

$= -16 - 52 - 34 + 12 = -90$ **Ans.**

$f(3) = 2(3)^3 - 13(3)^2 + 17(3) + 12$

$= 54 - 117 + 51 + 12 = 0$ **Ans.**

(e) Factorisation of an Expression Reducible to A Quadratic Expression :

Ex.17 Factorise :- $8 + 9(a - b)^6 - (a - b)^{12}$

Sol. $-8 + 9(a - b)^6 - (a - b)^{12}$

Let $(a - b)^6 = x$

Then $-8 + 9x - x^2 = -(x^2 - 9x + 8) = -(x^2 - 8x - x + 8)$

$= -(x - 8)(x - 1)$

$= -[(a - b)^6 - 8][(a - b)^6 - 1]$

$= [1 - (a - b)^6][(a - b)^6 - 8]$

$= [(1)^3 - \{(a - b)^2\}^3][\{(a - b)^2\}^3 - (2)^3]$

$= [1 - (a - b)^2][1 + (a - b)^4 + (a - b)^2][(a - b)^2 - 2][(a - b)^4 + 4 + 2(a - b)^2]$ **Ans.**

Ex.18 Factorise : $6x^2 - 5xy - 4y^2 + x + 17y - 15$

Sol. $6x^2 + x[1 - 5y] - [4y^2 - 17y + 15]$

$= 6x^2 + x[1 - 5y] - [4y^2 - 17y + 15]$

$= 6x^2 + x[1 - 5y] - [4y(y - 3) -5(y - 3)]$

$= 6x^2 + x[1 - 5y] - (4y - 5)(y - 3)$

$= 6x^2 + 3(y - 3)x - 2(4y - 5)x - (4y -)(y - 3)$

$= 3x[2x + y \ 3] - (4y - 5)(2x + y - 3)$

$= (2x + y - 3)(3x - 4y + 5)$ **Ans.**