5.2 EUCLID'S AXIOMS AND POSTULATES

A 'point', a 'line' and a 'plane' are the basic concepts to be used in geometry.

(a) Axioms :

The basic facts which are granted without proof are called axioms.

(b) Euclid's Definitions :

(i) A point is that which has not part.

(ii) A line is breathless length.

(iii) The ends of a line segment are points.

(iv) A straight line is that which has length only.

(v) A surface is that which has length and breadth only.

(vi) The edges of surface are lines.

(vii) A plane surface is that which lies evenly with the straight lines on itself.

(c) Euclid's Five Postulates :

(i) A straight line may be drawn from any one point to any other point.

(ii) A terminated line or a line segment can be produced infinitely.

(iii) A circle can be drawn with any centre and of any radius.

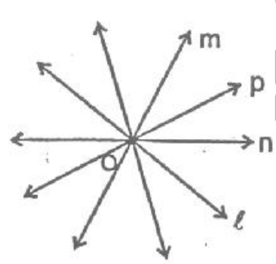
(iv) All right angles are equal to one another.

(v) If a straight line falling on two straight lines makes the exterior angles on the same side of it taken together less than two right angles, then the two straight lines if produced infinitely meet on that side on which the sum of angles are less than two right angles.

(d) Important Axioms :

(i) A line is the collection of infinite number of points.

(ii) Through a given point, an infinite lines can be drawn.



(iii) Given two distinct points, there is one and only one line that contains both the points.

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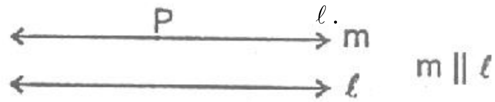
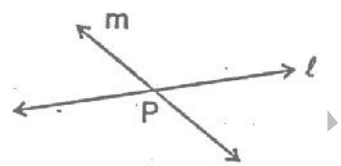
(iv) If P is a point outside a line ℓ, then one and only one line can be drawn through P which is parallel to ℓ.

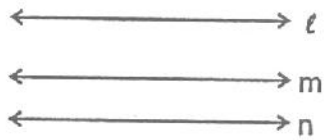


(v) Two distinct lines can not have more than one point in common.



(vi) Two lines which are both parallel to the same line, are parallel to each other.

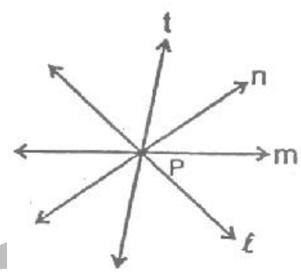
i.e. $\ell \| n, m \| n \Rightarrow \ell \| m$



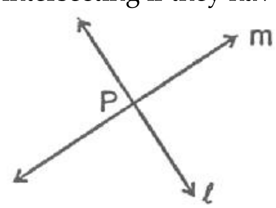
SOME IMPORTANT DEFINITOINS

(i) Collinear points : Three or more points are said to be collinear if there is a line which contains all of them.

(ii) Concurrent Lines : Three or more lines are said to be concurrent if there is a point which lies on all of them.



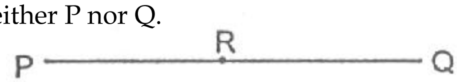
(iii) Intersecting lines : Two lines are intersecting if they have a common point. The common point is called the "point of intersection".



(iv) Parallel lines : Two lines *I* and *m* in a plane are said to be parallel lines if they do not have a common point.

(v) Line Segment : Given two points A and B on a line *l*, the connected part (segment) of the line with end points at A and B, is called the line segment AB.

(vi) Interior point of a line segment : A point R is called an interior point of a line segment PQ if R lies between P and Q but R is neither P nor Q.



(vii) Congruence of line segment : Two line segments AB and CD are congruent if trace copy of one can be superposed on the other so as to cover it completely and exactly in this case we write $AB \cong CD$. In other words we can say two lines are congruent if their lengths is same.

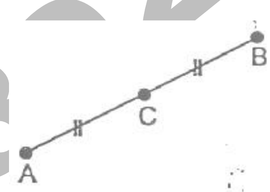
(viii) Distance between two points : The distance between two points P and Q is the length of line segment PQ

(ix) Ray : Directed line segment is called a ray. If AB is a ray then it is denoted by $\overline{AB}$. Point A is called initial point of ray.

(x) Opposite rays : Two rays AB and AC are said to be opposite rays if they are collinear and point A is the only common point of the two rays.

Ex.1 If a point C lies between two points A and B such that AC = BC, then prove that $AC = \frac{1}{2}$ AB. Explain by drawing the figure.

Sol. According to the given statement, the figure will be as shown alongside in which the point C lies between two points A and B such that AC = BC.



Clearly, AC + BC = AB

$\Rightarrow$ AC + AC = AB [$\because$ AC = BC]

$\Rightarrow$ 2AC = AB

And, $AC = \frac{1}{2} AB$

Ex.2 Give a definition for each of the following terms. Are there other terms that need to be defined first ? What are they, and how might you define them ?

(i) parallel lines (i) perpendicular lines (iii) line segment (iv) radius

Sol. **(i) Parallel lines :** Lines which don't intersect any where are called parallel lines.

(ii) Perpendicular lines : Two lines which are at a right angle to each other are called perpendicular lines.

(iii) Line segment : it is a terminated line.

(iv) Radius : The length of the line-segment joining the centre of a circle to any point on its circumference is called its radius.

Ex.3 How would you rewrite Euclid' fifth postulate so that it would be easier to understand ?

Sol. Two distinct intersecting lines cannot be parallel to the same line.

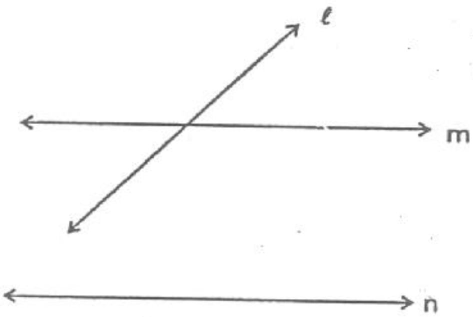
Ex.4 Does Euclid's fifth postulate imply the existence of parallel lines ? Explain.

Sol. if a straight line ℓ falls on two straight lines m and n such that sum of the interior angles on one side of ℓ is two right angles, then by Euclid's fifth postulate the line will not meet on this side of ℓ. Next, we know that the sum of the interior angles on the other side of line ℓ also be two right angles. Therefore they will not meet on the other side. So, the lines m and n never meet and are, therefore parallel.

Theorem 1 : If ℓ, m, n are lines in the same plane such that ℓ intersects m and n $\parallel$ m, then ℓ intersects n also.

Given : Three lines ℓ, m, n in the same plane s.t. ℓ intersects m and n $\parallel$ m.

To prove : Lines ℓ and n are intersecting lines.



Proof : Let ℓ and n be non intersecting lines. Then. $\ell \parallel n$.

But, n $\parallel$ m [Given]

$\therefore$ $\ell \parallel n$ and $n \parallel m \Rightarrow \ell \parallel m$

$\Rightarrow$ ℓ and m are non-intersecting lines.

This is a contradiction to the hypothesis that ℓ and m are intersecting lines.

So our supposition is wrong.

Hence, ℓ intersects line n.

Theorem 2 : If lines AB, AC, AD and AE are parallel to a line ℓ, then points A, B, C, D and E are collinear.

Given : Lines AB, AC, AD and AE are parallel to a line ℓ.

To prove : A, B, C, D, E are collinear.

Proof : Since AB, AC, AD and AE are all parallel to a line ℓ Therefore point A is outside ℓ and lines AB, AC, AD, AE are drawn through A and each line is parallel to ℓ.

But by parallel lines axiom, one and only one line can be drawn through the point A outside it and parallel to ℓ.

This is possible only when A, B, C, D, and E all lie on the same line. Hence, A, B, C, D and E are collinear.