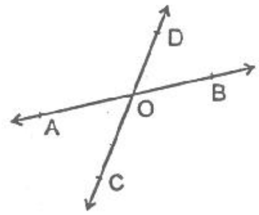
6.2 IMPORTANT THEOREMS

Theorem 1: If two lines intersect each other, then the vertically opposite angles are equal.

Given : Two lines AB and CD intersecting at a point O.



To prove : (i) $\angle AOC = \angle BOD$

(ii) $\angle BOC = \angle AOD$

Proof : Since ray OD stands on AB

$\therefore$ $\angle AOD + \angle DOB = 180^0$ ...(i) [linear pair]

again, ray OA stands on CD

$\therefore$ $\angle AOC + \angle AOD = 180^0$ ...(ii) [linear pair]

by (i) & (ii) we get

$\angle AOD + \angle DOB = \angle AOC + \angle AOD$

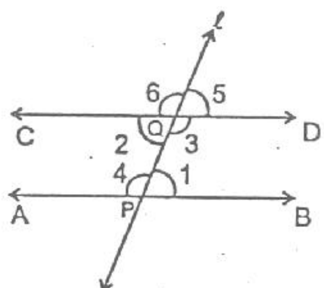
$\Rightarrow$ $\angle DOB = \angle AOC$

$\Rightarrow$ $\angle AOC = \angle DOB$

Similarly we can prove that $\angle BOC = \angle DOA$ Hence Proved.

Theorem 2 : If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

Given : AB and CD are two parallel lines, Transversal *l* intersects AB and CD at P and Q respectively making two pairs of alternate interior angles, $\angle 1$, $\angle 2$ & $\angle 3$, $\angle 4$.



To prove : $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Proof : Clearly, $\angle 2 = \angle 5$ [Vertically opposite angles]

And, $\angle 1 = \angle 5$ [Corresponding angles]

$\therefore$ $\angle 1 = \angle 2$

Also, $\angle 3 = \angle 6$ [Vertically opposite angles]

And, $\angle 4 = \angle 6$ [Corresponding angles]

$\therefore$ $\angle 3 = \angle 4$ **Hence, Proved.**

ILLUSTRATIONS

Ex.1 Two supplementary angles are in ratio 4 : 5, find the angles,

Sol. Let angles are 4x & 5x.

$\therefore$ Angles are supplementary

$\therefore$ $4x + 5x = 180^0 \Rightarrow 9x = 180^0$

$\Rightarrow$ $x = \dfrac{180^0}{9} = 20^0$

$\therefore$ Angles are $4 \times 20^0, 5 \times 20^0 \Rightarrow 80^0$ & 100^0 **Ans.**

Ex.2 If an angle differs from its complement by 10, find the angle.

Sol. let angles is x^0 then its complement is $90 - x^0$.

Now given $x^0 - (90 - x^0) = 10$

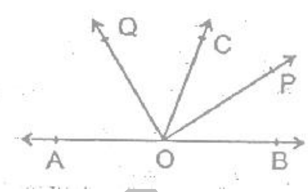
$\Rightarrow$ $x^0 - 90^0 + x^0 = 10$

$\Rightarrow$ $2x^0 = 10 + 90 = 100$

$\Rightarrow$ $x^0 = \dfrac{100^0}{2} = 50^0$

$\therefore$ Required angle is 50^0. **Ans.**

Ex.3 In figure, OP and OQ bisects $\angle BOC$ and $\angle AOC$ respectively. Prove that $\angle POQ = 90^0$.



Sol. $\therefore$ OP bisects $\angle BOC$

$\therefore$ $\angle POC = \dfrac{1}{2}\angle BOC$ ...(i)

Also OQ bisects $\angle AOC$

$\therefore$ $\angle COQ = \dfrac{1}{2}\angle AOC$ ...(ii)

$\therefore$ OC stands on AB

$\therefore$ $\angle AOC + \angle BOC = 180^0$ [Linear pair]

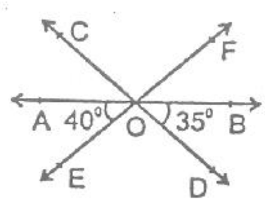
$\Rightarrow$ $\dfrac{1}{2}\angle AOC + \dfrac{1}{2}\angle BOC = \dfrac{1}{2} \times 180^0$

$\Rightarrow$ $\angle COQ + \angle POC = 90^0$ [Using (i) & (ii)]

$\Rightarrow$ $\angle POQ = 90^0$ [By angle sum property]

Hence Proved.

Ex.4 In figure, lines AB, CD and EF intersect at O. Find the measures of $\angle AOC$, $\angle DOE$ and $\angle BOF$



Sol. Given $\angle AOE = 40^0$ & $\angle BOD = 35^0$

Clearly $\angle AOC = \angle BOD$ [Vertically opposite angles]

$\Rightarrow \angle AOC = 35^0$ **Ans.**

$\angle BOF = \angle AOE$ [Vertically opposite angles]

$\Rightarrow \angle BOF = 40^0$ **Ans.**

Now, $\angle AOB = 180^0$ [Straight angles]

$\Rightarrow \angle AOC + \angle COF + \angle BOF = 180^0$ [Angles sum property]

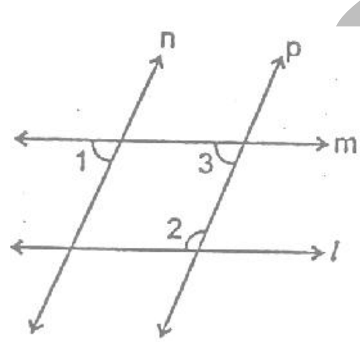
$\Rightarrow 35^0 + \angle COF + 40^0 = 180^0$

$\Rightarrow \angle COF = 180^0 - 75^0 = 105^0$

Now, $\angle DOE = \angle COF$ [Vertically opposite angles]

$\therefore \angle DOE = 105^0$ **Ans.**

Ex.5 In figure if $I \parallel m$, $n \parallel p$ and $\angle 1 = 85^0$ find $\angle 2$



Sol. $\therefore$ $n \parallel p$ and m is transversal

$\therefore \angle 1 = \angle 3 = 85^0$ [Corresponding angles]

Also $m \parallel I$ & p is transversal

$\therefore \angle 2 + \angle 3 = 180^0$ [$\because$ Consecutive interior angles]

$\Rightarrow \angle 2 + 85^0 = 180^0$

$\Rightarrow \angle 2 + 180^0 - 85^0$

$\Rightarrow \angle 2 = 95^0$ **Ans.**