7.3 SOME INEQUALITY RELATIONS IN A TRIANGLE

(i) If two sides of triangle are unequal, then the longer side has greater angle opposite to it. i.e. if in any ΔABCAB > AC then ∠C > ∠B.

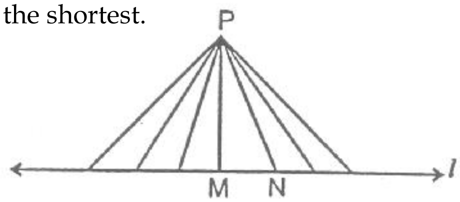
(ii) In a triangle the greater angle has the longer side opposite to it.

i.e. if in any ΔABC ∠A > ∠B then BC > AC.

(iii) The sum of any two sides of a triangle is greater than the third side.

i.e. if in any ΔABC, AB + BC > AC, BC + CA > AB and AC + AB > BC.

(iv) Of all the line segments that can be drawn to a given line, from a point, not lying on it, the perpendicular line segment is the shortest.

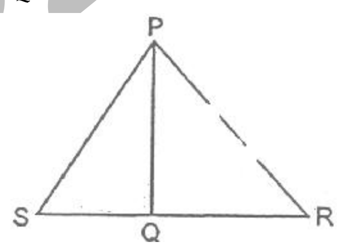


P is any point not lying online ℓ, PM ⊥ then PM < P N.

(v) The difference of any two sides of a triangle is less than the third side.

i.e. In any ΔABC, AB - BC < AC, BC - CA <AB and AC < AB < BC.

Ex.5 In figure, PQ = PR, show that PS > PQ



Sol. In ΔPQR

∴ PQ = PR

⇒ ∠PRQ = ∠PQR ....(i) [Angles opposite to equal sides]

In ΔPSQ, SQ is produced to R

∴ Ext. ∠PQR > ∠PSQ ....(ii)

∠PRQ > ∠PSQ [By eq. (i) and (ii)]

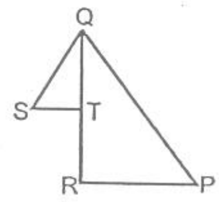
⇒ ∠PRS > ∠PSR

⇒ PS > PR [Sides opposite to greater angles is larger]

But, PR = PQ

∴ PS > PQ **Hence Proved.**

Ex.6 In figure, T is a point on side QR of ΔPQR and S is a point such that RT = ST. Prove that PQ + PR > QS



Sol. In ΔPQR we have

$PQ + PR > QR$

$\Rightarrow$ $PQ + PR > QT + TR$

$\Rightarrow$ $PQ + PR > QT + ST$ $\therefore RT = ST$

In ΔQST $QT + ST > SQ$

$\therefore$ $PQ + PR > SQ$

Hence Proved.