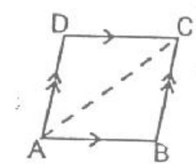
8.2 PARALLELOGRAM

A parallelogram is a quadrilateral in which both the pairs of opposite sides are parallel.

Theorem 1 : A diagonal of a parallelogram divides the parallelogram into two congruent triangles.

Given : A parallelogram ABCD.



To Prove : A diagonal divides the parallelogram into two congruent triangles

i.e., if diagonal AC is drawn then $\Delta ABC \cong \Delta CDA$

and if diagonal BD is drawn then $\Delta ABD \cong \Delta CDB$

Construction : Join A and C

Proof : Sine, ABCD is a parallelogram

$AB \parallel DC$ and $AD \parallel BC$

In ΔABC and ΔCDA

$\angle BAC = \angle DCA$ [Alternate angles]

$\angle BCA = \angle DAC$ [Alternate angles]

And, $AC = AC$ [Common side]

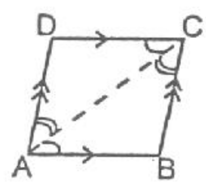
$\therefore$ $\Delta ABC \cong \Delta CDA$ [By ASA]

Similarly, we can prove that

$\Delta ABD \cong \Delta CDB$

Theorem 2 : In a parallelogram, opposite sides are equal.

Given : A parallelogram ABCD in which $AB \parallel DC$ and $AD \parallel BC$.



To Prove : Opposite sides are equal i.e., AB = DC and AD = BC

Construction : Join A and C

Proof : In ΔABC and ΔCDA

$\angle BAC = \angle DCA$ [Alternate angles]

$\angle BCA = \angle DAC$ [Alternate angles]

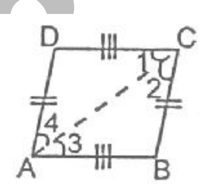
$AC = AC$ [Common]

$\therefore$ $\Delta ABC \cong \Delta CDA$ [By ASA]

$\Rightarrow$ AB = DC and AD = BC [By cpctc] **Hence Proved.**

Theorem 3: If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram

Given : A quadrilateral ABCD in which



To Prove: ABCD is a parallelogram i.e., AB ∥ DC and AD ∥ BC

Construction : Join A and C

Proof : In ΔABC and ΔCDA

AB = DC [Given]

AD = BC [Given]

And AC = AC [Common]

∴ ΔABC ≅ ΔCDA [By SSS]

⇒ ∠1 = ∠3 [By cpctc]

And ∠2 = ∠4 [By cpctc]

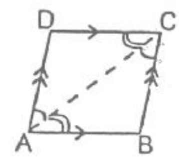
But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

∴ AB ∥ DC and AD ∥ BC

⇒ ABCD is a parallelogram. **Hence Proved.**

Theorem 4 : In a parallelogram, opposite angles are equal.

Given : A parallelogram ABCD in which AB ∥ DC and AD ∥ BC.



To Prove : Opposite angles are equal

i.e. ∠A = ∠C and ∠B = ∠D

Construction : Draw diagonal AC

Proof : In ΔABC and ΔCDA :

∠BAC = ∠DCA [Alternate angles]

∠BCA = ∠DAC [Alternate angles]

AC = AC [Common]

∴ ΔABC ≅ ΔCDA [By ASA]

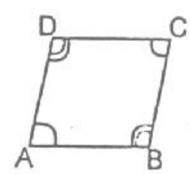
⇒ ∠B = ∠D [By cpctc]

And, ∠BAD = ∠DCB i.e., ∠A = ∠C

Similarly, we can prove that ∠B = ∠D **Hence Proved.**

Theorem 5: If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

Given : A quadrilateral ABCD in which opposite angles are equal.



i.e., ∠A = ∠C ad ∠B = ∠D

To prove : ABCD is a parallelogram i.e, AB ∥ DC and AD ∥ BC.

Proof : Since, the sum of the angles of quadrilateral is 360^0

⇒ $\angle A + \angle B + \angle C + \angle D = 360^0$

⇒ ∠A + ∠D + ∠A + ∠D = 360.. [∠A = ∠C and ∠B = ∠D]

⇒ $2\angle A = 2\angle D = 360^0$

⇒ $\angle A + \angle D = 180^0$ [Co-interior angle]

⇒ AB ∥ DC

Similarly,

$\angle A + \angle B + \angle C + \angle D = 360^0$

⇒ $\angle A + \angle B + \angle A + \angle B = 360^0$ [∠A = ∠C and ∠B = ∠D]

⇒ $2\angle A + 2\angle V = 360^0$

⇒ $\angle A + \angle B = 180^0$ [∵ This is sum of interior angles on the same side of transversal AB]

∴ AD ∥ BC

So, AB ∥ DC and AD ∥ BC

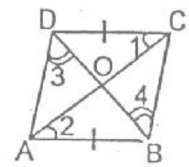
⇒ ABCD is a parallelogram. **Hence Proved.**

Theorem 6 : The diagonal of a parallelogram bisect each other.

Given : A parallelogram ABCD. Its diagonals AC and BD intersect each other at point O.

To Prove : Diagonals AC and BD bisect each other i.e., OA = OC and OB = OD.

Proof : In ΔAOB and ΔCOD



∵ AB ∥ DC and BD is a transversal line.

∴ ∠ABO = ∠DCO [Alternate angles]

∴ AB ∥ DC and AC is a transversal line.

∴ ∠BAO = ∠DCO [Alternate angles]

And, AB = DC

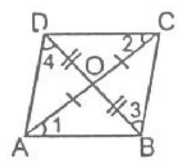
⇒ ΔAOB ≅ ΔCOD [By ASA]

⇒ OA = OC and OB = OD [By cpctc] **Hence Proved.**

Theorem 7 : If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Given : A quadrilateral ABCD whose diagonals AC and BD bisect each other at point O.

i.e., OA = OC and OB = OD



To prove : ABCD is a parallelogram

i.e., AB ∥ DC and AD ∥ BC.

Proof : In ΔAOB and ΔCOD

OA = OC [Given]

OB = OD [Given}

And, ∠AOB = ∠COD [Vertically opposite angles]

⇒ ΔAOB ≅ ΔCOD [By SAS]

⇒ ∠1 = ∠2 [By cpctc]

But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

∴ AB is parallel to DC i.e., AB ∥ DC

Similarly,

ΔAOD ≅ ΔCOB [By SAS]

⇒ ∠3 = ∠4

But these are also alternate angles ⇒ AD ∥ BC

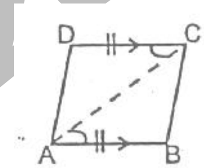
AB ∥ DC and AD ∥ BC ⇒ ABCD is parallelogram. **Hence Proved.**

Theorem 8 : A quadrilateral is a parallelogram, if a pair of opposite sides is equal and parallel.

Given : A quadrilateral ABCD in which AB ∥ DC and AB = DC.

To Prove : ABCD is a parallelogram

i.e., AB ∥ DC and AD ∥ BC.



Construction : Join A and C.

Proof : Since AB is parallel to DC and AC is transversal

∠BAC = ∠DCA [Alternate angles]

AB = DC [Given]

And AC = AC [Common side]

⇒ ΔBAC ≅ ΔDCA [By SAS]

⇒ ∠BCA = ∠DAC [By cpctc]

But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

⇒ AD ∥ BC

Now, AB ∥ DC (given) and AD ∥ BC [Proved above]

⇒ ABCD is a parallelogram **Hence Proved.**

REMARKS :

In order to prove that given quadrilateral is parallelogram, we have to prove that :

(i) Opposite angles of the quadrilateral are equal, or

(ii) Diagonals of the quadrilateral bisect each other, or

(iii) A pair of opposite sides is parallel and is of equal length, or

(iv) Opposite sides are equal.

(v) Every diagonal divides the parallelogram into two congruent triangles.