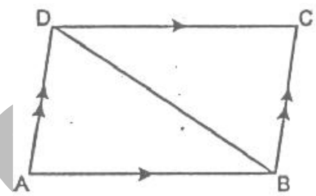
9.2 IMPORTANT THEOREMS

Theorem -1 A diagonal of parallelogram divides it into two triangles of equal area.

Given : A parallelogram ABCD whose one of the diagonals is BD.

To prove : ar (ΔABD) = ar (ΔCDB).

Proof : In ΔABD and ΔCDB.



AB = DC [Opp. sides of a $\|^{gm}$]

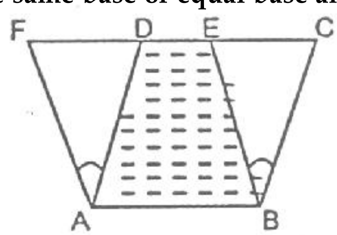
AD = BC [Opp. sides of a $\|^{gm}$]

BD = BD [Common side]

$\therefore$ $\Delta ABD \cong \Delta CDB$ [By SSS]

$\therefore$ ar (ΔABD) = ar(ΔCDB) [Congruent area axiom] **Hence Proved.**

Theorem -2: Parallelograms on the same base or equal base and between the same parallels are equal in area.



Given : Two $\|^{gm}$ ABCD and ABEF on the same base AB and between the same parallels AB and FC.

To Prove : ar($\|^{gm}$ ABCD) = ar($\|^{gm}$ ABEF)

Proof : In ΔADF and ΔBCE, we have

AD = BC [Opposite sides of a $\|^{gm}$]

AF = BE [Opposite sides of a $\|^{gm}$]

$\angle DAF = \angle CBE$ [$\because$ AD $\|$ BC and AF $\|$ BE]

[Angle between AD and AF = angle between BC and BE]

$\therefore$ $\Delta ADF \cong \Delta BCE$ [By SAS]

$\therefore$ ar(ΔADF) = ar(ΔBCE) ....(i)

$\therefore$ ar($\|^{gm}$ ABCD) = ar(ABED) + ar(ΔBCE)

= ar(ABED) + ar(ΔADF) [Using (i)]

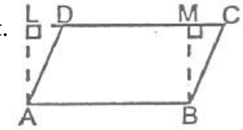
= ar($\|^{gm}$ ABEF).

Hence, ar($\|^{gm}$ ABCD) = ar($\|^{gm}$ ABEF). **Hence Proved.**

NOTE : A rectangle is also parallelogram.

Theorem -3: The are of parallelogram is the product of its base and the corresponding altitude.

Given : A $\|^{gm}$ ABCD in which AB is the base and AL is the corresponding height.



To prove : Area ($\|^{gm}$ ABCD) = AB × AL.

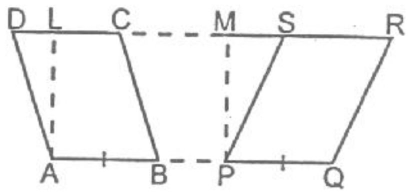
Construction : Draw BM $\perp$ DC so that rectangle ABML is formed.

Proof : $\|^{gm}$ ABCD and rectangle ABML are on the same base AB and between the same parallel lines AB and LC.

$\therefore$ ar($\|^{gm}$ ABCD) = ar(rectangle ABML) = AB × AL.

$\therefore$ area of a $\|^{gm}$ = base × height. **Hence Proved.**

Theorem-4 : Parallelograms on equal bases and between the same parallels are equal in area.



Given : Two $\|^{gm}$ ABCD and PQRS with equal base AB and PQ and between the same parallels, AQ and DR.

To prove: ar($\|^{gm}$ ABCD) = ar($\|^{gm}$ PQRS).

Construction : Draw AL $\perp$ DR and PM $\perp$ DR.

Proof : AB $\|$ DR, AL $\perp$ DR and PM $\perp$ Dr

$\therefore$ AL = PM.

$\therefore$ ar($\|^{gm}$ ABCD) = AB × AL

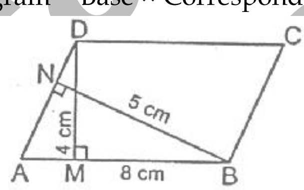
= PQ × PM [$\because$ AB = PQ and AL = PM]

= a($\|^{gm}$ PQRS). **Hence Proved.**

ILLUSTRATIONS :

Ex.1 In a parallelogram ABCD, AB = 8 cm. The altitudes corresponding to sides AB and AD are respectively 4 m and 5 cm. Find AD.

Sol. We know that, Area of a parallelogram = Base × Corresponding altitude



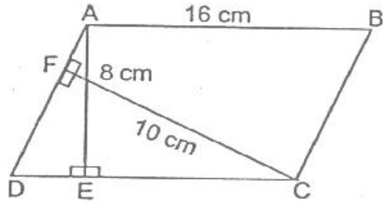
$\therefore$ Area of parallelogram ABCD = AD × BN = AB × DM

$\Rightarrow$ AD × 5 = 8 × 4

$\Rightarrow$ AD = $\frac{8 \times 4}{5}$

= 6.4 cm. **Ans.**

Ex.2 **In figure,** ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm find AD.



Sol. We have AB = 16 cm, AE = 8 cm CF = 10 cm.

We know that are of parallelogram = Base × Height [Base = CD, height = AE]

ABCD = CD × AE = 16 × 8 = 128 cm^2

Again, Area of parallelogram = Base × Height = AD × CF [Base = AD, height = CF]

128 = AD × 10

⇒ AD = $\frac{128}{10}$ =12.8 cm **Ans.**

Ex.3 ABCD is a quadrilateral and BD is one of its diagonal as shown in the figure. Show that the quadrilateral ABCD is a parallelogram and find its area.

Sol. From figure, the transversal DB is intersecting a pair of lines DC and AB such that

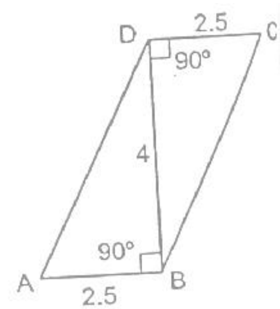
∠CDB = ∠ABD = 90^0.

Hence these angles from a pair of alternate equal angles.

∴ DC ∥ AB.

Also DC = AB = 2.5 units.

∴ Quadrilateral ABCD is a parallelogram.



Now, area of parallelogram ABCD

= Base × Corresponding altitude

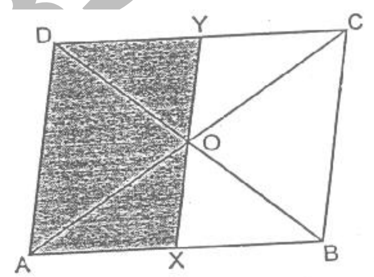
= 2.5 × 4

= 10 sq. units **Ans.**

Ex.4 The diagonals of a parallelogram ABCD intersect in O. A line through O meets AB is X and the opposite side CD in Y. Show that ar (quadrilateral AXYD) = $\frac{1}{2}$ far(parallelogram ABCD).

Sol. ∴ AC is a diagonal of the parallelogram ABCD.

ar(ΔACD) = $\frac{1}{2}$ ar(ABCD) ...(i)



Now, in Δs AOX and COY,

AO = CO

∵ Diagonals of parallelogram bisect each other.

∠AOX = ∠COY [Vert. opp. ∠s]

∠OAX = ∠OCY [Alt. Int. ∠s]

∴ AB ∥ DC and transversal AC intersects them

∴ ΔAOX ≅ ΔCOY [ASA]

∴ ar(ΔAOX) = ar(ΔCOY) ....(ii)

Sonu Sir 9582289230

Adding ar(quad. AOYD) to both sides of (ii), we get

ar(quad. AOYD) + ar(ΔAOX) = ar(quad. AOYD) + ar(ΔCOY)

$\Rightarrow$ ar(quad. AXYD) = ar(ΔACD) = $\frac{1}{2}$ ar($\parallel$gm ABCD) (using (i)) **Hence Proved.**