CHAPTER – 1
REAL NUMBER

1.1 EUCLID'S DIVISION LEMMA AND ALGORITHM:
EUCLID'S DIVISION LEMMA:

Let **'a'** and **'b'** be any two positive integers. Then, there exists unique integers **'q'** and **'r'** such that **a = b + r,** where **0 $\le$ r b.** If **b | a,** than **r = 0.**

Ex.1 Show that any positive odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5, where q is some integer.

Sol. Let 'a' be any positive integer and b = 6. Then, by Euclid's division lemma there exists integers 'a' and 'r' such that

a = 6q + r, where 0 $\le$ r < 6.

$\Rightarrow$ a = 6q or, a = 6q + 1 or, a = 6q + 2 or, a = 6a + 3 or, a = 6q + 4 or, a = 6q + 5.

[$\therefore$ 0 $\le$ r < 6 $\Rightarrow$ r = 0, 1,2,3,4,5]

$\Rightarrow$ a = 6q + 1 or, a = 6q + 3 or, a = 6q + 5.

[$\therefore$ a is an odd integer, $\therefore$ $\therefore$ 6q, a $\ne$ 6q + 2, a $\ne$ 6q + 4]

Hence, any odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5.

Ex.2 Use Euclid's Division Lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9 m + 8, for some integer q.

Sol, Let x be any positive integer. Then, it is of the form 3q or, 3q + 1 or, 3 + 2.

Case - I When x = 3q

$\Rightarrow$ $x^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$, where $m = 9q^3$

Case - II when x = 3q + 1

$\Rightarrow$ $x^3 = (3q + 1)^3$

$\Rightarrow$ $x^3 = 2q^3 + 27q^2 + 9q + 1$

$\Rightarrow$ $x^3 = 9q\,(3q^2 + 3q + 1) + 1$

$\Rightarrow$ $x^3 = 9m + 1$, where $m = q\,(3q^2 + 3q + 1)$.

Case -III when x = 3q + 2

$\Rightarrow$ $x^3 = (3q + 2)^3$

$\Rightarrow$ $x^3 = 27q^3 + 54q^2 + 36q + 8$

$\Rightarrow$ $x^3 = 9q(3q^2 + 6q + 4) + 8$

$\Rightarrow$ $x^3 = 9m + 8$, where $m = 3q^2 + 6q + 4)$

Hence, x^3 is either of the form 9m of 9m + 1 or 9m + 8.

Ex.3 Prove that the square of any positive integer of the form 5q + 1 is of the same form.

Sol. Let x be any positive's integer of the form 5q + 1.

When x= 5q + 1

$x^2 = 25q^2 + 10q + 1$

$x^2 = 5(5q + 2) + 1$

Let m = q (5q + 2).

$x^2 = 5m + 1.$

Hence, x^2 is of the same form i.e. 5m + 1.

EUCLID'S DIVISION ALGORITHM:

If **'a'** and **'b'** are positive integers such that **a = bq + r,** then every common divisor of **'a'** and **'b'** is a common divisor of **'b'** and **'r'** and vice-versa.

Ex.4 Use Euclid's division algorithm to find the H.C.F. of 196 and 38318.

Sol. Applying Euclid's division lemma to 196 and 38318.

38318 = 195 × 196 + 98

196 = 98 × 2 + 0

The remainder at the second stage is zero. So, the H.C.F. of 38318 and 196 is 98.

Ex.5 If the H.C.F. of 657 and 963 is expressible in the form 657x + 963 × (-15), find x.

Sol. Applying Euclid's division lemma on 657 and 963.

963 = 657 × 1 + 306

657 = 306 × 2 + 45

306 = 45 × 6 + 36

45 = 36 × 1 + 9

36 = 9 × 4 + 0

So, the H.C.F. of 657 and 963 is 9.

Given : 657x + 963 × (-15) = H.C.F. of 657 and 963.

657 x + 963 × (-15) = 9

657 x = 9 + 963 × 15

657 x = 14454

$$x = \frac{14454}{657} = 22.$$

Ex.6 What is the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively.

Sol. Clearly, the required number is the H.C.F. of the number 626 - 1 = 625, 3127 - 2 3125 and 15628 - 3 = 15625.

15628 - 3 = 15625.

Using Euclid's division lemma to find the H.C.F. of 625 and 3125.

3125 = 625 × 5 + 0

Clearly, H.C.F. of 625 and 3125 is 625.

Now, H.C.F. of 625 and 15625

15625 = 625 × 25 + 0

So, the H.C.F. of 625 and 15625 is 625.

Hence, H.C.F. of 625, 3125 and 15625 is 625.

Hence, the required number is 625.

Ex.7 144 cartons of coke cans and 90 cartons of Pepsi cans are to be stacked is a canteen. If each stack is of same height and is to contains cartons of the same drink, what would be the greatest number of cartons each stack would have ?

Sol. In order to arrange the cartons of the same drink is the same stack, we have to find the greatest number that divides 144 and 90 exactly. Using Euclid's algorithm, to find the H.C.F. of 144 and 90.

$144 = 90 \times 1 + 54$

$90 = 54 \times 1 + 36$

$54 = 36 \times 1 + 18$

$36 = 18 \times 2 + 0$

So, the H.C.F. of 144 and 90 is 18.

Number of cartons in each stack = 18.