1.3 CONTRADICTION METHOD

Ex.12 Prove that $\sqrt{2}$ is an irrational number.

Sol. Let assume on the contrary that $\sqrt{2}$ is a rational number.

Then, there exists positive integer a and b such that

$\sqrt{2} = \frac{a}{b}$ where, a and b are co primes i.e. their HCF is 1.

$\Rightarrow \quad (\sqrt{2})^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow \quad 2 = \frac{a^2}{b^2}$

$\Rightarrow \quad a^2 = 2b^2$

$\Rightarrow \quad a^2$ is multiple of 2

a is a multiple of 2 ....(i)

$\Rightarrow \quad$ a = 2c for some integer c.

$\Rightarrow \quad a^2 = 4c^2$

$\Rightarrow \quad 2b^2 = 4c^2$

$\Rightarrow \quad b^2 = 2c^2$

$\Rightarrow \quad b^2$ is a multiple of 2

b is a multiple of 2 ....(ii)

From (i) and (ii), a and b have at least 2 as a common factor. But this contradicts the fact that a and b are co-prime. This means that $\sqrt{2}$ is an irrational number.

Ex.13 Prove that $3 - \sqrt{5}$ is an irrational number.

Sol. Let assume that on the contrary that $3 - \sqrt{5}$ is rational.

Then, there exist co-prime positive integers a and b such that,

$3 - \sqrt{5} = \frac{a}{b}$

$\Rightarrow \quad 3 - \frac{a}{b} = \sqrt{5}$

$\Rightarrow \quad \frac{3b - a}{b} = \sqrt{5}$

$\Rightarrow \quad \sqrt{5}$ is rational [$\because$ a,b, are integer $\therefore \frac{3b-a}{b}$ is a rational number]

This contradicts the fact that $\sqrt{5}$ is irrational

Hence, $3 - \sqrt{5}$ is an irrational number.