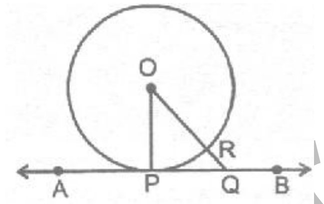
10.2 THEOREMS

THEOREM -1

Statement: **A tangent to a circle i perpendicular to the radius through the point of contact.**



Given : A circle C (O, r) and a tangent AB at a point P.

To prove : OP $\perp$ AB

Construction : Take any points Q, other than P on the tangent AB. Join OQ. Suppose OQ meets the circle at R.

Proof: Among all line segments joining the point O to a point on AB, the shorted one is perpendicular to AB. So, to prove that OP $\perp$ AB, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Clearly OP = OR

Now, OQ OR + RQ

$\Rightarrow$ OQ > OR

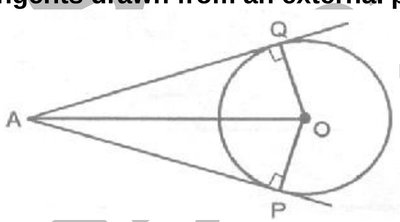
$\Rightarrow$ OQ > OP ($\therefore$ OP = OR)

Thus, OP is shorter than any other segment joining O to any point of AB.

Hence, OP $\perp$ AB.

THEOREM - 2

Statement : **Lengths of two tangents drawn from an external point to a circle are equal.**



Given: AP and AQ are two tangents drawn from a point A to a circle C (O, r).

To prove : AP = AQ

Construction : Join OP, OQ and OA.

Proof : In Δ AOQ and Δ APO

$\angle$OQA = $\angle$OPA [Tangent at any point of a circle is perp. to radius through the point of contact]

AO = AO [Common]

OQ = OP [Radius]

So, by R.H.S. criterion of congruency $\Delta AOQ \cong \Delta AOP$

$\therefore$ AQ = AP [By CPCT] Hence Proved.

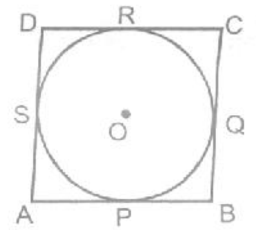
Result :

(i) If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre. $\angle$OAQ = $\angle$OAP [By CPCT]

(ii) If two tangents are drawn to a circle from an external point, they are equally inclined to the segment, joining the centre to that point $\angle$OAQ = $\angle$OAP [By CPCT]

Ex. 1 If all the sides of a parallelogram touches a circle, show that the parallelogram is a rhombus.

Sol. Given : Sides AB, BC, CD and DA of a $\|^{gm}$ ABCD touch a circle at P,Q,R and S respectively.



To prove $\|^{gm}$ ABCD is a rhombus.

Proof : AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

[Tangents drawn from an external point to a circle are equal]

Adding (1), (2), (3) and (4), we get

$\Rightarrow$ AP + BP + CR + DR = AS + BQ + CQ + DSZ

$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS_ + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ AB + AB = AD + AD [In a $\|^{gm}$ ABCD, opposite side are equal]

$\Rightarrow$ 2AB = 2AD or AB = AD

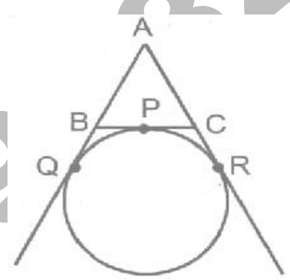
But AB = CD AND AD = BC [Opposite sides of a $\|$ gem]

$\therefore$ AB = BC = CD = DA

Hence, $\|^{gm}$ ABCD is a rhombus.

Ex.2 A circle touches the BC of a Δ ABC at P and touches AB and AC when produced at Q and R respectively as shown in figure, Show that $= \frac{1}{2}$ (Perimeter of Δ ABC).

So. **Given :** A circle is touching side BC of Δ ABC at P and touching AB and AC when produced at Q and R respectively.



To prove : $AQ = \frac{1}{2}$ (perimeter of Δ ABC)

Proof : AQ = AR ....(i)

BQ = BP ....(ii)

CP = CR ....(iii)

[Tangents drawn from and external point to a circle are equal]

Now, perimeter of Δ ABC = AB + BC + CA

= AB + BP + PC + CA

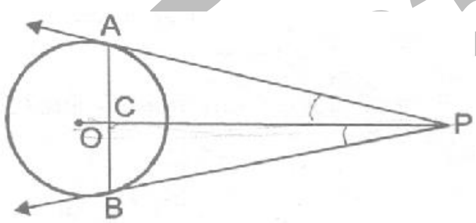
= (AB + BQ) + (CR + CA) [From (ii) and (iii)]

= AQ + AR = AQ + AQ [From (i)]

$AQ = \frac{1}{2}$ (perimeter of Δ ABC).

Ex.3 Prove that the tangents at the extremities of any chord make equal angles with the chord.

Sol. Let AB be a chord of a circle with centre O, and let AP and BP be the tangents at A and B respectively. Suppose, the tangents meet at point P. Join OP. Suppose OP meets AB at C.



We have to prove that

$\angle PAC = \angle PBC$

In triangles PCA and PCB

PA = PB [$\therefore$ Tangent from an external point are equal]

$\angle APC = \angle BPC$ [$\therefore$ PA and PB are equally inclined to OP]

And PC = PC [Common]

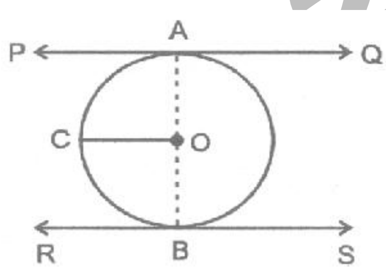
So, by SAS criteria of congruence

$\Delta PAC \cong \Delta BPC$

$\Rightarrow$ $\angle PAC = \angle PBC$ [By CPCT]

Ex.4 Prove that the segment joining the points of contact of two parallel tangents passes through the centre.

Sol. Let PAQ and RBS be two parallel tangents to a circle with centre O. Join OA and OB. Draw $OC \| PQ$ Now, $PA \| CO$



$\Rightarrow \quad \angle PAO + \angle COA = 180^0$ [Sum of co-interior angle is 180^0]

$\Rightarrow \quad 90^0 + \angle COA = 180^0$ [$\because \angle PAO = 90$]

$\Rightarrow \quad \angle COA = 90^0$

Similarly, $\angle CON = 90^0$

$\therefore \quad \angle COA + \angle COB = 90^0 + 90^0 = 180^0$

Hence, AOB is a straight line passing through O.