Chapter - 1

ASSIGNMENT

Q. 1. What is the difference between a collection and a set? Give reasons to support your answer?

Q. 2. Which of the following collections are sets? Justify your answer:

(i) A collection of all natural numbers less than 50.

(ii) The collection of good hockey players in India.

(iii) The collection of all girls in your class.

(iv) The collection of most talented writers of India.

(v) The collection of difficult topics in mathematics.

(vi) The collection of all months of a year beginning with the letter J.

(vii) A collection of novels written by Munshi Prem Chand.

(viii) The collection of all questions in this chapter.

(ix) A collection of most dangerous animals of the world.

(x) The collection of prime integers.

Q. 3. If A = {0,1,2,3,4,5,6, 7,8,9,10}, then insert the appropriate symbol $\in$ or $\notin$ in each of the following blank spaces:

(i) 4.....A (ii) -4.....A (iii) 12 ....A

(iv) 9 ....A (v) 0.....A (vi) -2.....A

Q.4. Describe the following sets in Roster form:

(i) {x: x is a letter before e in the English alphabet); (ii) $\{x \in N : x^2 < 25\}$;

(iii) $\{x \in N : x \text{ is a prime number, } 10 < x < 20\}$; (iv) $\{x \in N : x = 2n, n \in N\}$;

(v) $\{x \in R : x > x\}$. (vi) {x : x is a prime number which is a divisor of 60}

(vii) {x : x is a two digit number such that the sum of its digits is 8}

(viii) The set of all letters in the word 'Trigonometry'.

(ix) The set of all letters in the word 'Better'.

Q. 5. Describe the following sets in set-builder form:

(i) A = {1, 2,3,4,5, 6}; (ii) B = {1,1/2, 1/3, 1/4, 1/5,...};

(iii) C = {0,3, 6, 9,12,...}; (iv) D = {10,11,12,13,14,15};

(v) E = {0}; (vi) {1,4,9,16, ...,100}

(vii) {2,4,6,8,.....} (viii) {5,25,125,625}

Q. 6. List all the elements of the following sets:

(i) $A = \{x : x^2 \le 10, x \in Z\}$ (ii) $B = \{x : x = \frac{1}{2n-1}, 1 \le n \le 5\}$

(iii). C = {x : x is an integer, $-\frac{1}{2} < x < \frac{9}{2}$}

(iv) D = {x : x is a vowel in the word "EQUATION"}

(v) E = {x : x is a month of a year not having 31 days}

(vi) F = {x : x is a letter of the word "MISSISSIPPI"}

Q.7. Match each of the sets on the left in the roster form with the same set on the right described in the set-builder form:

(i) {A,P, L,E} (i) x : x + 5 = 5,x ∈ Z}

(ii) {5, - 5} (ii) {x : x is a prime natural number and a divisor of 10}

(iii) {0} (iii) \x : x is a letter of the word "RAJASTHAN"}

(iv) {1, 2, 5,10,} (iv) {x : x is a natural number and divisor of 10}

(v) {A, H. 1, R, S, T, N} (v) x : x^2 - 25 = 0}

(vi) {2, 5} (vi) {x: x is a letter of the word "APPLE"}

Q.8. Write the set of all vowels in the English alphabet which precede q.

Q. 9. Write the set of all positive integers whose cube is odd.

Q.10. Write the set $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$ in the set-builder form.

Q. 11. Which of the following are examples of empty set ?

(i) Set of all even natural numbers divisible by 5;

(ii) Set of all even prime numbers;

(iii) {x: x^2 - 2 = 0 and x is rational};

(iv) {x: x is a natural number, x < 8 and simultaneously x > 12};

(v) (x: x is a point common to any two parallel lines}.

Q.12. Which of the following sets are finite and which are infinite ?

(i) Set of concentric circles in a plane;

(ii) Set of letters of the English Alphabets;

(iii) {x ∈ N : x > 5}; (iv) {x ∈ N : x < 200};

(v) {x ∈ Z : x < 5); (vi) {x ∈ R : 0 < x < 1}.

Q.13 Which of the following sets are equal ?

(i) A = {1, 2, 3}; (ii) B = {x ∈ R : x^2 - 2x + 1 = 0};

(iii) C = {1,2,2,3}; (iv) D = {x ∈ R : x^3 - $6x^2$ + 11x - 6 = 0}.

Q.14 Are the following sets equal ?

A = {x : x is a letter in the word reap}; B = {x : x is a letter in the word paper};

C = {x: x is a letter in the word rope},

Q.15 From the sets given below, pair the equivalent sets:

A = {1, 2,3}, B = {t, p, q, r, s}, C = {α, β, γ}, D = {a, e, i, o, u}.

Q.16 Are the following pairs of sets equal ? Give reasons.

(i) A = {2, 3}, B = {x : x is a solution of $x^2 + 5x + 6 = 0$};

(ii) A = {x : x is a letter of the word "WOLF"}; B = {x : x is a letter of the word "FOLLOW"}.

Q.17 From the sets given below, select equal sets and equivalent sets.

A = {0, a},B = {1,2,3,4} C = {4,8,12}, D = {3,l,2,4},

E = {1, 0}, F = {8,4,12} G = {1,5;7,11},H = {a,b}.

Q.18 Which of the following sets are equal?

A = {x : x ∈ N, x < 3}, B = {1, 2} C = {3, 1} D = {x: x ∈ N, x is odd, x < 5},

E = {1,2,1,1} F = {1,1,3}.

Q.19 Show that the set of letters needed to spell "CATARACT" and the set of letters needed to spell "TRACT" are equal.

Q.20 Which of the following statements are true ? Give reason to support your answer

(i) For any two sets A and B either A ⊆ B or B ⊆ A;

(ii) Every subset of an infinite set is infinite;

(iii) Every subset of a finite set is finite;

(iv) Every set has a proper subset;

(v) {a, b, a, b, a, b,...} is an infinite set;

(vi) {a, b, c} and {1,2,3} are equivalent sets;

(vii) A set can have infinitely many subsets.

Q.21 State whether the following statements are true or false:

(i) 1 ∈ {1,2,3} (ii) a ⊂ {b, c, a} (iii) {a} ∈ {a, b, c}

(iv) {a, b} = {a, a, b, b, a} (v) The set {x : x + 8 = 8} is the null set.

Q.23 Decide among the following sets, which are subsets of which:

A = {x : x satisfies $x^2 - 8x + 12 = 0$}, B = {2,4,6}, C = {2,4,6,8,....}, D = {6}.

Q.24 Write which of the following statements are true ? Justify your answer.

(i) The set of all integers is contained in the set of all rational numbers.

(ii) The set of all crows is contained in the set of all birds.

(iii) The set of all rectangles is contained in the set of all squares.

(iv) The set of all real numbers is contained in the set of all complex numbers.

(v) The sets P = {a} and B = {{a}} are equal.

(vi) The sets A = {x : x is a letter of the word "LITTLE"} and,

B = {x : x is a letter of the word "TITLE"} are equal.

Q.25 Which of the following statements are correct? Write a correct form of each of the incorrect statements.

(i) a ⊂ {a, b, c} (ii) {a} ∈ {a, b, c} (iii) a ∈ {{a}, b}

(iv) {a} ⊂ {{a),b} (v) {b, c} ⊂ {a, {b, c}} (vi) {a, b} ⊂ {a, {b, c}}

(vii) $\phi \in \{a, b\}$ (viii) $\phi \subset \{a, b, c\}$ (ix) $\{x : x + 3 = 3\} = \phi$

Q.26 Let A = {a, b, {c, d},e}. Which of the following statements are false and why?

(i) $\{c . d\} \subset A$ (ii) $\{c, d\} \in A$ (iii) $\{\{c, d\}\} \subset A$ (iv) $a \in A$

(v) $a \subset A$ (vi) $\{a, b, e\} \subset A$ (vii) $\{a, b, e\} \in A$ (viii) $\{a, b, c\} \subset A$

(ix) $\phi \in A$ (x) $\{\phi\} \subset A$

Q.27 Let A = {{1, 2, 3}, {4, 5}, {6, 7, 8}}. Determine which of the following is true or false:

(i) $1 \in A$ (ii) $\{1,2,3\} \subset A$ (iii) $\{6, 7,8\} \in A$

(iv) $\{\{4, 5\}\} \subset A$ (v) $\phi \in A$ (vi) $\phi \subset A$

Q.28. If A and B are two sets such that $A \subset B$. then find:

(i) $A \cap B$ (ii) $A \cup B$

Q.29 If A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8}, C = {7, 8, 9, 10,11} and D = {10, 11, 12,13,14}, find;

(i) $A \cup B$ (ii) $A \cup C$ (iii) $B \cup C$

(iv) $B \cup D$ (iv) $A \cup B \cup C$ (vi) $A \cup B \cup D$

(vii) $B \cup C \cup D$ (viii) $A \cap (B \cup C)$ (ix) $(A \cap B) \cap (B \cap C)$

(x) $(A \cup D) \cap (B \cup C)$.

Q.30 Let $A = (x : x \in N\}$, $B = \{x : x = 2n, n \in N\}$, $C = \{x : x = 2n -1, n \in N)$ and D = {x: x is a prime natural number!. Find:

(i) $A \cap B$ (ii) $A \cap C$ (iii) $A \cap D$

(iv) $B \cap C$ (v) $B \cap D$ (vi) $C \cap D$

Q.31. Let A = {3, 6,12,15,18, 21}, B = {4, 8, 12, 16, 20}, C = {2, 4, 6, 8,10,12,14,16} and D = {5,10,15, 20}. Find:

(i) A - B (ii) A - C (iii) A - D

(iv) B - A (v) C - A (vi) D - A

(vii) B - C (viii) B - D

Q.32 Let U = {1, 2, 3,4, 5, 6, 7, 8,9}, A = {1,2, 3, 4}, B = {2,4, 6, 8} and C = {3,4, 5, 6}. Find

(i) A' (ii) B' (iii) $(A \cap C)'$

(iv) $(A \cup B)'$ (v) $(A')'$ (vi) $(B - C)'$

Q.33 Let U = {1, 2, 3, 4, 5, 6, 7, 8,9}, A = {2, 4, 6, 8} and B = {2, 3,5, 7}. Verify that

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.

Q.34 Find the smallest set A such that $A \cup \{1,2\} = \{1,2,3,5,9\}$.

Q.35 Let A = {1,2,4,5} B = {2,3,5,6} C = {4,5,6,7}. Verify the following identities:

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iii) $A \cap (B - C) = (A \cap B) - (A \cap C)$ (iv) $A - (B \cup C) = (A - B) \cap (A - C)$

(v) $A - (B \cap C) = (A - B) \cup (A - C)$ (vi) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.

Q.36 If U = {2,3,5,7,9} is the universal set and A = {3, 7}, B = {2,5,7,9}, then prove that:

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.

Q.37 For any two sets A and B, show that the following statements are equivalent:

(i) $A \subset B$ (ii) $A - B = 0$ (iii) $A \cup B = B$ (iv) $A \cap B = A$.

Q.38 For three sets A, B and C, show that

(i) $A \cap B = A \cap C$ need not imply $B = C$ (ii) $A \subset B \Rightarrow C - B \subset C - A$

Q.39 For any two sets, prove that:

(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$

Q.40 Find sets A, B and C such that $A \cap B$, $A \cap C$ and $B \cap C$ are non-empty sets and

$A \cap B \cap C = \phi$.

Q.41 For any two sets A and B, prove that : $A \cap B = \phi \Rightarrow A \subseteq B'$.

Q.42 If A and B are sets, then prove that $A - B$, $A \cap B$ and $B - A$ are pairwise disjoint.

Q.43 Using properties of sets, show that for any two sets A and B,

$(A \cup B) \cap (A \cap B') = A$

Q.44 For any two sets of A and B, prove that:

(i) $A' \cup B = U \Rightarrow A \subset B$ (ii) $B' \subset A' \Rightarrow A \subset B$

Q.45 Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

Q.46 Show that for any sets A and B,

(i) $A = (A \cap B) \cup (A - B)$ (ii) $A \cup (B - A) = A \cup B$

Q.47 For any two sets A and B, prove that: $A' - B' = B - A$

Q.48 For any two sets A and B, prove the following :

(i) $A \cap (A' \cup B) = A \cap B$ (ii) $A - (A - B) = A \cap B$

(iii) $A \cap (A \cup B)' = \phi$ (iv) $A - B = A \Delta (A \cap B)$.

Q.49 If A, B, C are three sets such that $A \subset B$, then prove that

$C - B \subset C - A$

Q.50 A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, find the value of x.

Ans. $39 \leq x \leq 63$

Q.50 In a class of 35 students, 17 have taken mathematics, 10 have taken mathematics but not economics. Find the number of students who have taken both mathematics and economics and the number of students who have taken economics but not mathematics, if it is given that each student has taken either mathematics or economics or both.

Ans. 18

Q.51 In a town of '10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three news papers, find the number of families which buy (i) A only (ii) B only (iii) none of A, B and C.

Ans. (i) 3300 (ii) 1400 (iii) 4000

Q.52 A college awarded 38 medals in Football, 15 in Basketball and 20 to Cricket. If, these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports ? Ans. 9

Q.53. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects.Find the number of students that had

(i) only chemistry. (ii) only mathematics. (iii) only physics. (iv) physics and chemistry but not mathematics. (v) mathematics and physics but not chemistry. (vi) only one of the subjects. (vii) at least one of the three subjects. (viii) none of the subjects.

Ans. (i) 5 (ii) 4 (iii) 2 (iv) 1 (v) 6 (vi) 11 (vii) 23 (viii) 2

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

Q.1. For any set A, (A')' is equal to

(a) A' (b) A (c) ϕ (d) none of these.

Q.2. Let A and B be two sets in the same universal set. Then, A - B =

(a) $A \cap B$ (b) $A' \cap B$ (c) $A \cap B'$ (d) none of these

Q.3. The number of subsets of a set containing n elements is

(a) n (b) $2^n - 1$ (c) n^2 (d) 2^n

Q.4. For any two sets A and B, $A \cap (A \cup B)$ =

(a) A (b) B (c) ϕ (d) none of these.

Q. 5. if A = {1, 3, 5, B} and B = {2, 4}, then

(a) $4 \in A$ (b) $\{4\} \subset A$ (c) $B \subset A$ (d) none of these

Q.6. The symmetric difference of A and B is

(a) $(A - B) \cap (B - A)$ (b) $(A - B) \cup (B - A)$

(c) $(A \cup B) - (A \cap B)$ (d) $\{(A \cup B) - A\} \cup \{(A \cup B) - B\}$

Q. 7. The symmetric difference of A = {1, 2,3} and B = {3,4, 5} is

(a) {1,2} (b) {1,2,4,5} (c) {4,3} (d) {2,5,1,4,3}

Q.8. For any two sets A and B, $(A - B) \cup (B - A)$ =

(a) $(A - B) \cup A$ (b) $(B - A) \cup B$ (c) $(A \cup B) - (A \cap B)$ (d) $(A \cup B) \cap (A \cap B)$

Q. 9. Which of the following statement is false :

(a) $A - B = A \cap B'$ (b) $A - B = A - (A \cap B)$

(c) A - B = A - B' (d) A - B = (A ∪ B) - B

Q.10. For any three sets A, B and C

(a) A ∩ (B - C) = (A ∩ B) - (A ∩ C) (b) A ∩ (B - C) = (A ∩ B) - C

(c) A ∪ (B - C) = (A ∪ B) ∩ (A ∪ C) (d) A ∪ (B - C) = (A ∪ B) - (A ∪ C).

Q. 11. Let A = {x : x ∈ R, x ≥ 4) and B = {x ∈ R : x < 5}. Then, A ∩ B =

(a) (4,5) (b) (4,5) (c) (4,5) (d) (4,5)

Q.12. Let U be the universal set containing 700 elements. If A, B are sub-sets of U such that n (A) = 200, n (B) = 300 and n (A ∩ B) = 100. Then, n (A' ∩ B') =

(a) 400 (b) 600 (c) 300 (d) none of these.

Q.13. Let A and B be two sets such that n (A) = 16, n (B) = 14, n (A ∪ B) = 25. Then, n (A ∩ B) is equal to

(a) 30 (b) 50 (c) 5 (d) none of these

Q.14. If A = {1, 2, 3,4, 5}, then the number of proper subsets of A is

(a) 120 (b) 30 (c) 31 (d) 32

Q.15. In set-builder method the null set is represented by

(a) { } (b) Φ (c) {x :x ≠ x} (d) {x:x = x}

Q.16. If A and B are two disjoint sets, then n (A ∪ B) is equal to

(a) n(A) + n(B) (b) n (A) + n(B) - n(A ∩ B)

(c) n(A) + p'B) + n(A ∩ B) (d) n(A) n(B) (e) n (A) - n (B)

Q.17. For two sets A ∪ B = A iff

(a) B ⊆ A (b) A ⊆ B (c) A ≠ B (d) A = B

Q.18. If A and B are two sets such that n (A) = 70, n (B) = 60, n (A ∪ B) = 110, then n (A ∩ B) is equal to

(a) 240 (b) 50 (c) 40 (d) 20

Q.19. If A and B are two given sets, then A ∩ $(A \cap B)^c$ is equal to

(a) A (b) B (c) Φ (d) $A^c \cap B^c$

Q.20. If A - {x: x is a multiple of 3} and, B = {x: x is a multiple of 5}, then A - B is

(a) A ∩ B (b) A ∩ $\overline{B}$ (c) $\overline{A} \cap \overline{B}$ (d) $\overline{A \cap B}$

Q.21. In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then person travelling by car or bus is

(a) 80% (b) 40% (c) 60% (d) 70%

Q.22. If A ∩ B = B, then

(a) A ⊂ B (b) B ⊂ A (c) A = Φ (d) B = Φ

Q.23. An investigator interviewed 100 students to determine the performance of three drinks: milk, coffee and tea, The investigator reported that 10 students take all three drinks milk, coffee and tea;

20 students take milk and coffee; 25 students take milk and tea; 12 students take milk only; 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any of three drinks is

(a) 10 (b) 20 (c) 25 (d) 30

Q. 24. Two finite sets have m and n elements. The number of elements in the power set of first set is 48 more than the total number of elements in power set of the second set. Then, the values of m and n are:

(a) 7,6 (b) 6,3 (c) 6,4 (d) 7,4 (e) 3,7

Q. 25. In a class of 175 students the following data shows the number of students opting one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. How many students have offered Mathematics alone ?

(a) 35 (b) 48 (c) 60 (d) 22 (e) 30

ANSWERS OF MCQ

1. (b) 2. (c) 3. (d) 4. (a) 5. (d) 6. (b) 7. (b) 8. (c) 9. (c) 10. (a),(b),(c) 11. (c) 12. (c) 13. (c) 14. (c) 15. (c) 16. (a) 17. (a) 18. (d) 19. (d) 20. (b) 21. (c) 22. (b) 23. (b) 24. (c) 25. (c)