Chapter - 10

ASSIGNMENT

1 Find the angle between the lines joining the points (0, 0), (2, 3) and the points (2, –2), (3, 5).

2 What is the value of y so that the line through (3, y) and (2, 7) is parallel to the line through (–1, 4) and (0, 6)?

3 If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the line $\frac{1}{2}$, find the slope of the other line.

4 A ray of light passing through the point (1, 2) reflects on the x–axis at point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.

5 If points (a, 0), (0, b) and (x, y) are collinear, using the concept of slope prove that $\frac{x}{a}+\frac{y}{b}=1$.

6 Prove that a triangle which has one of the angle as 30°, cannot have all vertices with integral coordinates.

7 The vertices if a triangle are A $(x_1, x_1 \tan\theta_1)$, B$(x_2, x_2 \tan\theta_2)$ and C$(x_3, x_3 \tan\theta_3)$. If the circumcentre of Δ ABC coincides with the origin and H $(\bar{x}, \bar{y})$ is the orthocenter, show that

$$\frac{\bar{y}}{x}=\frac{\sin\theta_1+\sin\theta_2+\sin\theta_3}{\cos\theta_1+\cos\theta_2+\cos\theta_2}$$

8 What can be said regarding a line if its slope is
(i) zero (ii) positive (iii) negative?

9 Without using Pythagoras theorem, show that the points A (0, 4), B (1,2) and C (3,3) are the vertices of a right angled triangle.

10 If three points A (h, 0), P (a, b) and B (0, k) lie on a line, show that: $\frac{a}{h}+\frac{b}{k}=1$.

11 Write down the equations of the following lines:
(i) x–axis (ii) y–axis
(iii) A line parallel to x–axis at a distance of 3 units below x–axis.

(iv) Line parallel to y–axis at a distance of 5 units on the left hand side of it.

12 Find the equation of a line which is parallel to y–axis passes through (–4, 3).

13 Find the equation of a line which is parallel to x–axis and passes through (3, –5).

14 Write the value of $\theta \in \left(0, \frac{\pi}{2}\right)$ for which area of the triangle formed by points O (0, 0), A (a cos θ, b sin θ) and B (a cos θ, - b sin θ) is maximum.

15 Draw the lines x = -3,x = 2,y = - 2, y = 3 and write the coordinates of the vertices of the square so formed.

16 Find the equation of a line which is equidistant from the lines x = - 2 and x = 6.

17 Find the equation of the straight line intersecting y-axis at a distance of 2 units above the origin and making an angle of 30° with the positive direction of the x-axis.

18 Determine the equation of line through the point (–4, –3) and parallel to x–axis.

19 Find the equation of the line for which $\tan\theta = \frac{1}{2}$, where θ is the inclination of the line and (i) x–intercept equal to 4. (ii) y–intercept is $-\frac{3}{2}$.

20 Two lines passing through the point (2, 3) intersect each other at an angle of 60°. If slope of one line is 2, find the equation of the other line.

21 One side of a square makes an angle α with x–axis and one vertex of the square is at the origin. Prove that the equations of its diagonals are x (sin α + cos α) = y (cos α – sin α) and x (cos α – sin α) + y (sin α + cos α) = a, where a is the length of the side of the square.

22 Find the equations of the altitudes of the triangle whose vertices are A (7, –1), B(–2, 8) and C (1, 2).

23 The mid–points of the sides of a triangle are (2, 1), (–5, 7) and (–5, –5). Find the equations of the triangle.

24 Find the equation of the perpendicular bisector of the line segment joining the point (1, 1) and (2, 3).

25 Show that the perpendicular draw from the point (4, 1) on the line segment joining (6, 5) and (2, –1) divides it internally in the ratio 8 : 5.

26 Find the coordinates of the vertices of a inscribed in the triangle with vertices A (0, 0), B (2, 1) and C (3, 0); given that two of its vertices are on the side AC.

27 A line is such that its segment between the line $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation .

28 Find the equations to the diagonals of the rectangle the equations of whose sides are $x = a$, $x = a'$, $y = b$ and $y = b'$.

29 Find the equation of the straight line which

(i) makes equal intercepts on the axes and passes through the point (2, 3).

(ii) passes through the point (–5, 4) and is such that the portion intercepted between the axes is divided by the point in the ratio 1 : 2.

30 Find the equation of the line which cuts iff equal positive intercepts from the axes and passes through the point (α, β).

31 A straight line cuts intercepts from the axes of coordinates the sum of whose reciprocals is a constant. Show that. Show that it always passes through a fixed point.

32 A straight line passes through the point (α, β) and this point bisects the portion of the line intercepted between the axes. Show that the equation of the straight line is $\frac{x}{2\alpha} + \frac{y}{2\beta} = 1.$

33 Find the equation of the line, which passes through P (1, - 7) and meets the axes at A and B respectively so that 4AP - 3BP = 0.

34 Find the equations of the straight lines each of which passes through the point (3, 2) and cuts off intercepts a and b respectively on X and Y-axes such that a - b = 2.

35 Find the equation to the straight line which cuts off equal positive intercepts on the axes and their product is 25.

36 A straight canal is $4\frac{1}{2}$ miles from a place and the shortest route from this place to the canal is exactly north–east. A village is 3 mile north and four miles east from the place. Does it lie by the nearest edge of the canal?

37 If the straight line though the point P (3, 4) makes an angle $\frac{\pi}{6}$ with x–axis and meets the line 12 x + 5 y + 10 = 0 at Q, find the length of PQ.

38 The line joining two points A (2, 0), B(3, 1) is rotated about A in anti–clockwise direction through an angle of 15°. Find the equation of the line in the new position. If B goes to C in the new position, what will be the coordinates of C?

39 Find the equation of the line passing through the point (2, 3) and making an intercept of length 3 units between the line y + 2x = 2 and y + 2x = 5.

40 Find the distance of the point (2, 5) from the line 3x + y + 4 = 0 measured parallel to the line 3x - 4y + 8 = 0.

41 Find the distance of the point (3, 5) from the line 2x + 3y = 14 measured parallel to the line x - 2y = 1.

42 A straight line drawn through the point A (2, 1) making an angle $\pi/4$ with positive x-axis intersects another line x + 2y + 1 = 0 in the point B. Find length AB.

43 Reduce the equation $\sqrt{3}$ x + y + 2 = 0 to:
(i) slope-intercept form and find slope and y-intercept;
(ii) intercept form and find intercept on the axes;
(iii) the normal form and find p and α.

44 Transform the equation of the line $\sqrt{3}$ x + y – 8 = 0 to (i) slope intercept form and find its slope and y–intercept (ii) intercept form and find intercepts on the coordinates axes (iii) normal form and find the inclination of the perpendicular segment from the origin on the line with the axis and its length.

45 Reduce the line 3 x – 4y +4 = 0 and 4x – 3y + 12 = 0 to the normal form and hence determine which line is near to the origin.

46 Prove that the slope of a line is invariant under the translation of the axes.

47 The line 2x – y = 5 turns about the point on it, whose ordinate and abscissa are equal, through an angle of 45 in the anti–clockwise direction. Find the equation of the line in the new position.

48 Find the area of the triangle formed by the lines y = x, y = 2x and y = 3x + 4.

49 Find the value of m for which the lines mx + (2m + 3) y + m + 6 = 0 and (2m + 1) x + (m – 1) y + m – 9 = 0 intersect at a point on Y–axis.

50 Write the area of the figure formed by the lines a | x | + b | y | + c = 0.

51 If the lines x + ay + a = 0, bx + y + b = 0 and cx + cy + l = 0 are concurrent, then write the value of 2abc - ab - bc - ca.

52 Show that the area of the triangle formed by the lines $y = m_1 x$, $y = m_2 x$ and y = c is equal to $\frac{c^2}{4}(\sqrt{33}+\sqrt{11})$, where m_1, m_2 are the roots of the equation $x^2 + (\sqrt{3}+2)x+\sqrt{3}-1=0$.

53 Find the area of the triangle formed by the line $y = m_1 x + c_1$, $y = m_2 x + c_2$ and x = 0.

54 Find the area of the triangle formed by the lines y = x, y = 2x and y = 3x + 4.

55 Find the equation of the line parallel to Y–axis and drawn through the point of intersection of the line x – 7y + 5 = 0 and 3x + y = 0.

56 Two consecutive sides of a parallelogram are 4x + 5y = 0 and 7x + 2y = 0. If the equation of one diagonal is 11x + 7y = 9, find the equation of the other diagonal.

57 Prove that the lines $y = \sqrt{3}x + 1$, y = 4 and $y = -\sqrt{3}x + 2$ form an equilateral triangle.

58 For what value of λ are the three lines 2x - 5y + 3 = 0, 5x - 9y + λ = 0 and x - 2y + 1 = 0 concurrent?

59 Find the value of λ, if the lines 3x – 4y – 13 = 0, 8x – 11y – 33 = 0 and 2x – 3y + λ = 0 are concurrent.

60 If the lines $a_1 x + b_1 y + 1 = 0$, $a_2 x + b_2 y + 1 = 0$ and $a_3x + b_3y + 1 = 0$ are concurrent, show that the points (a_1, b_1), (a_2, b_2) and (a_3, b_3) are collinear.

61 If a, b, c are in A.P., prove that the straight lines ax + 2 y + 1 = 0, bx + 3 y +1 = 0 and cx + 4 y + 1 = 0 are concurrent.

62 If the lines $ax + y + 1 = 0$, $x + by + 1 = 0$ and $x + y + c = 0$ are concurrent ($a \neq b \neq c \neq 1$), prove that

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

63 Find the angles between each of the following pairs of straight lines:

(i) $3x + y + 12 = 0$ and $x + 2y - 1 = 0$ (ii) $3x - y + 5 = 0$ and $x - 3y + 1 = 0$

(iii) $(m^2 - mn)\, y = (mn + n^2)\, x + n^3$ and $(mn + m^2)\, y - (mn - n^2)\, x + m^3$.

64 Find the angle between the line joining the points (2, 0), (0, 3) and the line $x + y = 1$

65 Are the points (3, 4) and (2, – 6) on the same or opposite sides of the line $3x – 4y = 8$?

66 If the points (4, 7) and ($\cos\theta$, $\sin\theta$), where $0 < \theta < \pi$, lie on the same side of the line $x + y – 1 = 0$, then prove that θ lies in the first quadrant.

67 Determine value of α for which the point (α, α^2) lies inside the triangle formed by the lines $2x + 3y – 1 = 0$, $x + 2y – 3 = 0$ and $5x – 6y – 1 = 0$.

68 If p is the length of the perpendicular from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$, then prove that

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

69 What are the points on x–axis whose perpendicular distance from the line $4x + 3y = 12$ is 4?

70 Find the equation of the straight line which cuts off intercept on X-axis which is twice that on Y–axis and is at a unit distance from the origin.

71 If the length of the perpendicular from the point (1, 1) to the line $ax - by + c = 0$ be unity, show that $\frac{1}{c}+\frac{1}{a}-\frac{1}{b}=\frac{c}{2ab}$.

72 Show that the product of perpendiculars on the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ from the Points $(\pm\sqrt{a^2-b^2}, 0)$ is b^2.

73 What are the points on y-axis whose distance from the line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units?

74 Prove that the parallelogram formed by the lines

$$\frac{x}{a}+\frac{y}{b}=1,\frac{x}{b}+\frac{y}{a}=1,\frac{x}{a}+\frac{y}{b}=2 \text{ and } \frac{x}{b}+\frac{y}{a}=2 \text{ is a rhombus.}$$

75 Find the equation of the line mid way between the parallel line $9x + 6y – 7 = 0$ and $3x + 2y + 6 = 0$.

76 Find the equations of the two straight lines through (7, 9) and making an angle of 60° with the line $x – \sqrt{3}\, y – 2\sqrt{3} = 0$.

77 A vertex of an equilateral triangle is (2, 3) and the opposite side $x + y = 2$. Find the equations of the sides.

78 Two sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point (1, - 10). Determine the equation of the third side.

79 The equation of the base of an equilateral triangle is $x + y = 2$ and its vertex is (2, -1). Find the length and equations of its sides.