10.6 Angle between two non-parallel lines

If θ be the angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ and intersecting at A. Then, $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$. If θ is angle between two lines, then $\pi - \theta$ is also the angle between them.

(1) **Angle between two straight lines when their equations are given :** The angle θ between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by, $\tan\theta = \left|\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right|$.

(2) **Conditions for two lines to be coincident, parallel, perpendicular and intersecting :** Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are,

(a) Coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(b) Parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(c) Intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(d) Perpendicular, if $a_1a_2 + b_1b_2 = 0$

Equation of straight line through a given point making a given angle with a given line

The equation of the straight lines which pass through a given point (x_1, y_1) and make a given angle α with given straight line $y = mx + c$ are, $y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$.

Equations of the bisectors of the angles between two straight lines

The equation of the bisectors of the angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given by,

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}} \qquad .....(i)$$

Algorithm to find the bisector of the angle containing the origin : Let the equations of the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. To find the bisector of the angle containing the origin, we proceed as follows:

Step I : See whether the constant terms c_1 and c_2 in the equations of two lines positive or not. If not, then multiply both the sides of the equation by –1 to make the constant term positive.

Step II : Now obtain the bisector corresponding to the positive sign *i.e.*, $\dfrac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\dfrac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$.

This is the required bisector of the angle containing the origin.

The bisector of the angle containing the origin means the bisector of the angle between the lines which contains the origin within it.

(1) **To find the acute and obtuse angle bisectors :** Let θ be the angle between one of the lines and one of the bisectors given by (i). Find $\tan\theta$. If $|\tan\theta|<1$, then this bisector is the bisector of acute angle and the other one is the bisector of the obtuse angle.

If $|\tan\theta|>1$, then this bisector is the bisector of obtuse angle and other one is the bisector of the acute angle.

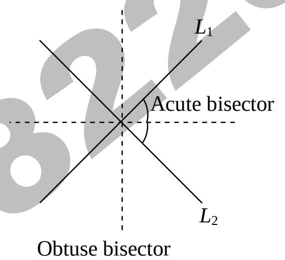
(2) Method to find acute angle bisector and obtuse angle bisector

(i) Make the constant term positive, if not.

(ii) Now determine the sign of the expression $a_1a_2+b_1b_2$.

(iii) If $a_1a_2+b_1b_2>0$, then the bisector corresponding to "+" sign gives the obtuse angle bisector and the bisector corresponding to "–" sign is the bisector of acute angle between the lines.

(iv) If $a_1a_2+b_1b_2<0$, then the bisector corresponding to "+" and "–" sign given the acute and obtuse angle bisectors respectively.



Bisectors are perpendicular to each other.

If $a_1a_2+b_1b_2>0$, then the origin lies in obtuse angle and if $a_1a_2+b_1b_2<0$, then the origin lies in acute angle.