11.2 Various Terms

Definition

The curves obtained by intersection of a plane and a double cone in different orientation are called conic section.

Definitions of various important terms

(1) **Focus:** The fixed point is called the focus of the conic-section.

(2) **Directrix:** The fixed straight line is called the directrix of the conic section.

(3) **Eccentricity:** The constant ratio is called the eccentricity of the conic section and is denoted by e.

(4) **Axis:** The straight line passing through the focus and perpendicular to the directrix is called the axis of the conic section. A conic is always symmetric about its axis.

(5) **Vertex:** The points of intersection of the conic section and the axis are called vertices of conic section.

(6) **Centre :** The point which bisects every chord of the conic passing through it, is called the centre of conic.

(7) **Latus-rectum:** The latus-rectum of a conic is the chord passing through the focus and perpendicular to the axis.

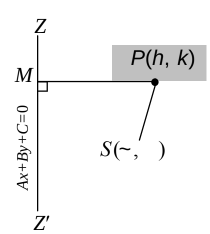
(8) **Double ordinate:** The double ordinate of a conic is a chord perpendicular to the axis.

(9) **Focal chord:** A chord passing through the focus of the conic is called a focal chord.

(10) **Focal distance:** The distance of any point on the conic from the focus is called the focal distance of the point.

General equation of a conic section when its focus, directrix and eccentricity are given

Let $S(\sim, \ )$ be the focus, $Ax + By + C = 0$ be the directrix and e be the eccentricity of a conic. Let $P(h,k)$ be any point on the conic. Let PM be the perpendicular from P, on the directrix. Then by definition,



$$SP = ePM \implies SP^2 = e^2 PM^2$$

$$\implies (h - \sim)^2 + (k - \ )^2 = e^2 \left(\frac{Ah + Bk + C}{\sqrt{A^2 + B^2}} \right)^2$$

Thus the locus of (h,k) is $(x - \sim)^2 + (y - \ )^2 = e^2 \frac{(Ax + By + C)^2}{(A^2 + B^2)}$,

which is general equation of second degree.

Recognisation of conics

The equation of conics is represented by the general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ......(i)

and discriminant of above equation is represented by Δ, where $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

Case I : When $\Delta = 0$.

In this case equation (i) represents the degenerate conic whose nature is given in the following table.

Table : 18.1

S. No.	Condition	Nature of conic
1.	$\Delta = 0$ and $ab - h^2 = 0$`	A pair of coincident straight lines
2.	$\Delta = 0$ and $ab - h^2 < 0$	A pair of intersecting straight lines
3.	$\Delta = 0$ and $ab - h^2 > 0$	A point

Case II : When $\Delta \neq 0$.

In this case equation (i) represents the non-degenerate conic whose nature is given in the following table.

Table : 18.2

S. No.	CONDITION	Nature of conic
1.	$\Delta \neq 0, h = 0, a = b$, $e = 0$	A circle
2.	$\Delta \neq 0, ab - h^2 = 0$, $e = 1$	A parabola
3.	$\Delta \neq 0, ab - h^2 > 0$, $e < 1$	An ellipse
4.	$\Delta \neq 0, ab - h^2 < 0$, $e > 1$	A hyperbola
5.	$\Delta \neq 0, ab - h^2 < 0$, $a + b = 0$, $e = \sqrt{2}$	A rectangular hyperbola