11.3 Parabola

Definition

A parabola is the locus of a point which moves in a plane such that its distance from a fixed point (i.e., focus) in the plane is always equal to its distance from a fixed straight line (i.e., directrix) in the same plane.

Standard equation of the parabola

Let S be the focus, zz' be the directrix of the parabola and (x,y) be any point on parabola, then standard form of the parabola is $y^2 = 4ax$.

Some other standard forms of parabola are
(i) Parabola opening to left *i.e,* $y^2 = -4ax$
(ii) Parabola opening upwards *i.e.,* $x^2 = 4ay$
(iii) Parabola opening downwards *i.e.,* $x^2 = -4ay$
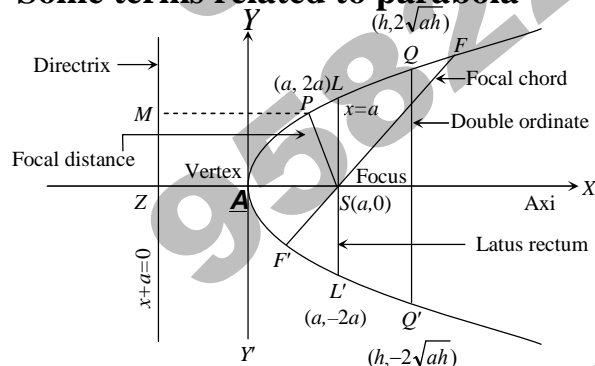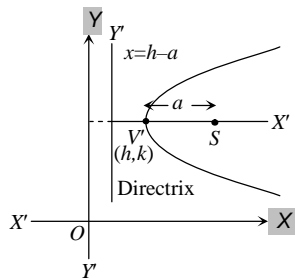
Some terms related to parabola

Table : 18.3

Important terms	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Focus	(a, 0)	(–a, 0)	(0, a)	(0, –a)
Directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Latusrectum	$4a$	$4a$	$4a$	$4a$
Focal distance $P(x,y)$	$x + a$	$a - x$	$y + a$	$a - y$

Special form of parabola $(y - k)^2 = 4a(x - h) = a$

The equation of a parabola with its vertex at (h, k) and axis as parallel to x-axis is $(y - k)^2 = 4a(x - h)$.

If the vertex of the parabola is (p, q) and its axis is parallel to y-axis, then the equation of the parabola is $(x - p)^2 = 4b(y - q)$.

Parametric equations of a parabola

Table : 18.4

Parabola	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Parametric Co-ordinates	$(at^2, 2at)$	$(-at^2, 2at)$	$(2at, at^2)$	$\left(2at, -at^2\right)$
Parametric Equations	$x = at^2$ $y = 2at$	$x = -at^2$ $y = 2at$	$x = 2at$ $y = at^2$	$x = 2at$, $y = -at^2$

The parametric equations of parabola $(y - k)^2 = 4a(x - h)$ are $x = h + at^2$ and $y = k + 2at$.

Position of a point and a line with respect to a parabola

(1) **Position of a point with respect to a parabola :** The point $P(x_1, y_1)$ lies outside, on or inside the parabola $y^2 = 4ax$ according as $y_1^2 - 4ax_1 >, =, < 0$.
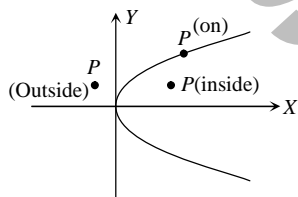
(2) **Intersection of a line and a parabola:** The line $y = mx + c$ does not intersect, touches or intersect a parabola $y^2 = 4ax$, according as $c >, =, < \dfrac{a}{m}$.

Condition of tangency : The line $y = mx + c$ touches the parabola, if $c = a/m$.