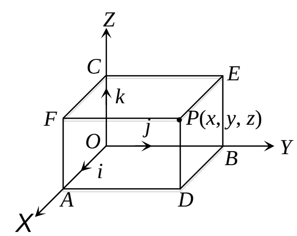
CHAPTER – 12
Co-ordinate Geometry of Three Dimensions

12.1 Introduction

Co-ordinates of a point in space

(1) **Cartesian co-ordinates :** Let *O* be a fixed point, known as origin and let *OX*, *OY* and *OZ* be three mutually perpendicular lines, taken as *x*-axis, *y*-axis and *z*-axis respectively, in such a way that they form a right-handed system.



The planes *XOY*, *YOZ* and *ZOX* are known as *xy*-plane, *yz*-plane and *zx*-plane respectively.

Also, $OA = x, OB = y, OC = z$.

The three co-ordinate planes (*XOY*, *YOZ* and *ZOX*) divide space into eight parts and these parts are called octants.

Sign of co-ordinates of a point : The signs of the co-ordinates of a point in three dimension follow the convention that all distances measured along or parallel to *OX*, *OY*, *OZ* will be positive and distances moved along or parallel to *OX′*, *OY′*, *OZ′* will be negative.

and $¤ = \tan^{-1}(y/x)$.

Cylindrical co-ordinates of

$P \equiv (u, ¤, z)$

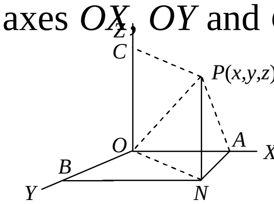
Distance formula

(1) **Distance formula:** The distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$.

(2) **Distance from origin:** Let *O* be the origin and *P*(*x*, *y*, *z*) be any point, then $OP = \sqrt{(x^2 + y^2 + z^2)}$.

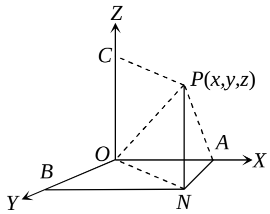
(3) **Distance of a point from co-ordinate axes:** Let $P(x, y, z)$ be any point in the space. Let PA, PB and PC be the perpendiculars drawn from P to the axes OX, OY and OZ respectively.



Then, $PA = \sqrt{(y^2 + z^2)}$

$PB = \sqrt{(z^2 + x^2)}$

$PC = \sqrt{(x^2 + y^2)}$



Section formula

(1) **Section formula for internal or external division:** Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points. Let R be a point on the line segment joining P and Q such that it divides the join of P and Q internally or externally in the ratio $m_1 : m_2$.

Then the co-ordinates of R are

$$\left(\frac{m_1x_2 \pm m_2x_1}{m_1 \pm m_2}, \frac{m_1y_2 \pm m_2y_1}{m_1 \pm m_2}, \frac{m_1z_2 \pm m_2z_1}{m_1 \pm m_2}\right).$$

(2) **Co-ordinates of the general point :** The co-ordinates of any point lying on the line joining points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ may be taken as $\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}, \frac{kz_2 + z_1}{k+1}\right)$, which divides PQ in the ratio $k : 1$. This is called general point on the line PQ.