12.4 Line

Straight line in space

Every equation of the first degree represents a plane. Two equations of the first degree are satisfied by the co-ordinates of every point on the line of intersection of the planes represented by them.

Therefore, the two equations of that line $ax+by+cz+d=0$ and $a'x+b'y+c'z+d'=0$ together represent a straight line.

(1) **Equation of a line passing through a given point**

Cartesian equation of a straight line passing through a fixed point (x_1, y_1, z_1) and having direction ratios *a*, *b*, *c* is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$.

(2) **Equation of line passing through two given points**

If $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ be two given points, the equations to the line *AB* are $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$.

Changing unsymmetrical form to symmetrical form

The unsymmetrical form of a line $ax+by+cz+d=0,\ \ a'x+b'y+c'z+d'=0$ can be changed to symmetrical form as follows : $\frac{x-\frac{bd'-b'd}{ab'-a'b}}{bc'-b'c}=\frac{y-\frac{da'-d'a}{ab'-a'b}}{ca'-c'a}=\frac{z}{ab'-a'b}$.

Intersection of two lines

Determine whether two lines intersect or not. In case they intersect, the following algorithm is used to find their point of intersection.

Algorithm:

Let the two lines be $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ …..(i)

and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ …..(ii)

Step I : Write the co-ordinates of general points on (i) and (ii). The co-ordinates of general points on (i) and (ii) are given by $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}=ª$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}=«$ respectively. *i.e.*, $(a_1ª+x_1, b_1ª+y_1+c_1ª+z_1)$

and $(a_2«+x_2, b_2«+y_2, c_2«+z_2)$.

Step II : If the lines (i) and (ii) intersect, then they have a common point.

$a_1ª+x_1=a_2«+x_2, b_1ª+y_1=b_2«+y_2$

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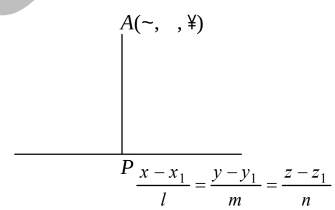
and $c_1ª + z_1 = c_2« + z_2$.

Step III : Solve any two of the equations in ª and « obtained in step II. If the values of ª and « satisfy the third equation, then the lines (i) and (ii) intersect, otherwise they do not intersect.

Step IV : To obtain the co-ordinates of the point of intersection, substitute the value of ª (or «) in the co-ordinates of general point (*s*) obtained in step I.

Foot of perpendicular from a point to the line

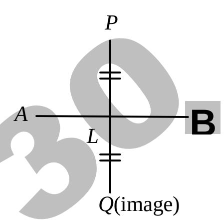
Foot of perpendicular from a point $A(~, \ , ¥)$ to the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$: If *P* be the foot of perpendicular, then *P* is $(lr + x_1, mr + y_1, nr + z_1)$. Find the direction ratios of *AP* and apply the condition of perpendicularity of *AP* and the given line. This will give the value of *r* and hence the point *P*, which is foot of perpendicular.



Length and equation of perpendicular : The length of the perpendicular is the distance *AP* and its equation is the line joining two known points *A* and *P*.

The length of the perpendicular is the perpendicular distance of given point from that line.

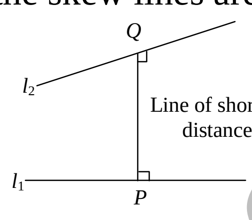
Reflection or image of a point in a straight line: If the perpendicular *PL* from point *P* on the given line be produced to *Q* such that *PL* = *QL*, then *Q* is known as the image or reflection of *P* in the given line. Also, *L* is the foot of the perpendicular or the projection of *P* on the line.



Shortest distance between two straight lines

(1) **Skew lines:** Two straight lines in space which are neither parallel nor intersecting are called skew lines.

Thus, the skew lines are those lines which do not lie in the same plane.



(2) **Line of shortest distance:** If l_1 and l_2 are two skew lines, then the straight line which is perpendicular to each of these two non-intersecting lines is called the "Line of shortest distance."

There is one and only one line perpendicular to each of lines l_1 and l_2.

(3) **Shortest distance between two skew lines**

Let two skew lines be, $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$

and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$

Therefore, the shortest distance between the lines is given by

$$d=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2-m_2n_1)^2+(n_1l_2-l_1n_2)^2+(l_1m_2-m_1l_2)^2}}.$$