Chapter - 13

ASSIGNMENT

1. Evaluate the left hand and right hand limits of the function defined by

$$f(x)=\begin{cases}1+x^2, \text{if } 0\le x\le 1\\ 2-x, \text{if } x>1\end{cases} \text{ at } x=1.$$

Also, show that $\lim_{x\to 1} f(x)$ does not exist.

2. If $f(x)=\begin{cases}5x-4, & 0<x\le 1\\ 4x^3-3x, & 1<x<2\end{cases}$, show that $\lim_{x\to 1} f(x)$ exists.

3. Find the left hand right hand limits of the greatest integer function f(x)= [x] = greatest integer less than or equal to x, at x = k, where k is an integer. Also show than f(x) does not exist.

4. Let f(x) be a function defined by f(x) = $\begin{cases}4x-5, & \text{if } x\le 2\\ x-\lambda, & \text{if } x>2\end{cases}$. Find λ, if $\lim_{x\to 2} f(x)$ exists.

5. If f(x) = $\begin{cases}mx^2+n, x<0\\ nx+m, 0\le x\le 1\\ nx^3+m, x>1\end{cases}$

For what of integers m, n does the limits $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ exist.

6. If f (x) = $\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$ For what value (s) of a does $\lim_{x\to a} f(x)$ exist?

7. Find k so that $\lim_{x\to 2} f(x)$ may exist, where f(x) = $\lim_{x\to -2}\begin{cases}2x+3, & x\le 2\\ x+k & x>2\end{cases}$.

8. Let f(x) be a function defined by f(x) = $\lim_{x\to 0}\begin{cases}\frac{3x}{|x|+2x}, & x\ne 2\\ 0, & x=2\end{cases}$.

9. Find $\lim_{x\to 5/2}[x]$.

10. Find $\lim_{x\to 3^+}\frac{x}{[x]}$. Is it equal to $\lim_{x\to 3^-}\frac{x}{[x]}$.

11. Evaluate: $\lim_{x\to 2}\frac{x^2-5x+6}{x^2-4}$.

12. Evaluate : $\lim_{x\to 2}\frac{x^3-3x^2+4}{x^4-8x^2+16}$.

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13. Evaluate: $\lim_{x\to 2}\frac{x^3-6x^2+11x-6}{x^2-6x+8}$.

14. Evaluate : $\lim_{x\to 1/2}\frac{8x^3-1}{16x^4-1}$.

15. Evaluate: $\lim_{x\to 4}\frac{(x^2-x-12)^{18}}{(x^3-8x^2+16x)^9}$.

16. Evaluate: $\lim_{x\to\sqrt{2}}\frac{x^9-3x^8+x^6-9x^4-4x^2-16x+84}{x^5-3x^4-4x+12}$.

17. $\lim_{x\to\sqrt{3}}\frac{x^2-3}{x^2+3\sqrt{3}x-12}$

18. $\lim_{x\to 1}\frac{x^4-3x^3+2}{x^3-5x^2+3x+1}$

19. Evaluate: $\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}$.

20. Evaluate: $\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-\sqrt{a^2-x^2}}{x^2}$.

21. Evaluate: $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$.

22. Evaluate: $\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$.

23. Evaluate: $\lim_{x\to\sqrt{10}}\frac{\sqrt{7-2x}-(\sqrt{5}-\sqrt{2})}{x^2-10}$.

24. $\lim_{x\to 7}\frac{x-5}{\sqrt{6x-5}-\sqrt{4x+5}}$

25. $\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$

26. Evaluate: $\lim_{x\to 2}\frac{x^{10}-1024}{x-2}$.

27. Evaluate: $\lim_{x\to a}\frac{x\sqrt{x}-a\sqrt{a}}{x-a}$.

28. Evaluate: $\lim_{x\to a}\frac{(x+2)^{5/3}-(a+2)^{5/3}}{x-a}$.

29. If $\lim_{x\to 2}\frac{x^n-2^n}{x-2}=80$ and $n\in N$, find n.

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30. Find the value of k, if $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$.

31. If $\lim_{x\to a}\frac{x^9+a^9}{x+a}=9$, find the value of a.

32. If $\lim_{x\to a}\frac{x^3-a^3}{x-a}=\lim_{x\to 1}\frac{x^4-1}{x-1}$, find all possible value of a.

33. Evaluate: $\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+...+(x+100)^{10}}{x^{10}+10^{10}}$

34. Evaluate : $\lim_{n\to\infty}\frac{1+2+3+...+n}{n^2}$.

35. Evaluate: $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$.

36. Evaluate: $\lim_{n\to\infty}\frac{n!}{(n+1)!-n!}$

37. $\lim_{x\to\infty}\frac{x}{\sqrt{4x^2+1}-1}$

38. $\lim_{n\to\infty}\frac{n^2}{1+2+3+....+n}$

39. $\lim_{x\to\infty}\frac{3x^{-1}+4x^{-2}}{5x^{-1}+6x^{-2}}$

40. $\lim_{x\to\infty}\frac{\sqrt{x^2+a^2}-\sqrt{x^2+b^2}}{\sqrt{x^2+c^2}-\sqrt{x^2+d^2}}$

41. $\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+...+\frac{n-1}{n^2}\right)$

42. $\lim_{n\to\infty}\frac{1^3+2^3+...+n^3}{(n-1)^4}$

43. $\lim_{n\to\infty}\frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$

44. $\lim_{x\to 0}\frac{\sin x^0}{3x}$

45. Evaluate the following limits:

(i) $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$ (ii) $\lim_{x\to 0}\frac{1-\cos 2x}{x}$ (iii) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ (iv) $\lim_{x\to 0}\frac{1-\cos 2mx}{1-\cos 2nx}$ (iv) $\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}$

46. Evaluate the following limits:

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(i) $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$ (ii) $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}$

47. Evaluate: $\lim_{x\to 0}\frac{\tan x+4\tan 2x-3\tan 3x}{x^2\tan x}$

48. Evaluate: $\lim_{x\to 0}\frac{\sin x-2\sin 3x+\sin 5x}{x}$

49. Evaluate: $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

50. Evaluate: $\lim_{x\to \pi/6}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$

51. Evaluate: $\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x^3}$

52. Evaluate: $\lim_{x\to 0}\frac{1-\cos\ x\ \cos\ 2x\ \cos\ 3x}{\sin^2 2x}$

53. Evaluate the following limits:

(i) $\lim_{x\to \frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}$

(ii) $\lim_{x\to \frac{\pi}{4}}\frac{\sin x-\cos x}{x-\frac{\pi}{4}}$

(iii) $\lim_{x\to \frac{\pi}{2}}\frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)}$

(i v) $\lim_{x\to \frac{\pi}{6}}\frac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}$

(v) $\lim_{x\to \pi/6}\frac{\sqrt{3}\sin x-\cos x}{x-\frac{\pi}{6}}$

54. Evaluate the following limits:

(i) $\lim_{x\to \pi/2}\frac{\cos x}{\frac{\pi}{2}-x}$ (ii) $\lim_{x\to a}\frac{\cos x-\cos a}{\cot x-\cot a}$

(iii) $\lim_{x\to \infty} x\tan\left(\frac{1}{x}\right)$ (iv) $\lim_{x\to a}\frac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}$

(v) $\lim_{x\to \pi/2}(\sec x-\tan x)$ vi) $\lim_{x\to \frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$

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