13.4 Differentiation

Introduction

The rate of change of one quantity with respect to some another quantity has a great importance.

The rate of change of a quantity '*y*' with respect to another quantity '*x*' is called the derivative or differential coefficient of *y* with respect to *x*.

Some standard differentiation

(1) Differentiation of algebraic functions: $\frac{d}{dx}x^n = nx^{n-1}$

In particular

(i) $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x)$ (ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

(iii) $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$

(2) Differentiation of trigonometric functions:

(i) $\frac{d}{dx}\sin x = \cos x$ (ii) $\frac{d}{dx}\cos x = -\sin x$

(iii) $\frac{d}{dx}\tan x = \sec^2 x$ (iv) $\frac{d}{dx}\sec x = \sec x \tan x$

(v) $\frac{d}{dx}\text{cosec } x = -\text{cosec } x \cot x$ (vi) $\frac{d}{dx}\cot x = -\text{cosec}^2 x$

Theorems for differentiation

Let $f(x), g(x)$ and $u(x)$ be differentiable functions

(1) If at all points of a certain interval, $f'(x) = 0$, then the function $f(x)$ has a constant value within this interval.

(2) Chain rule

(i) **Case I :** If *y* is a function of *u* and *u* is a function of *x*, then derivative of *y* with respect to *x* is $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ or $y = f(u) \Rightarrow \frac{dy}{dx} = f'(u)\frac{du}{dx}$.

(ii) **Case II :** If *y* and *x* both are expressed in terms of *t*, *y* and *x* both are differentiable with respect to *t*, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

(3) Sum and difference rule: Using linear property $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

(4) Product rule

(i) $\frac{d}{dx}(f(x).g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

(ii) $\frac{d}{dx}(u.v.w.) = u.v.\frac{dw}{dx} + v.w.\frac{du}{dx} + u.w.\frac{dv}{dx}$

(5) Scalar multiple rule: $\frac{d}{dx}(k\,f(x)) = k\frac{d}{dx}f(x)$

(6) Quotient rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$

Provided $g(x) \neq 0$.