Chapter - 2

ASSIGNMENT

1 Let A = {1, 2, 3} and B = {x : x $\in$ N, x is prime less than 5}. Find A × B and B × A.

2 If A = {1, 2}, form the set A × A × A.

3 Express A = {(a,b) : 2a + b - 5, a, b $\in$ W} as the set of ordered pairs.

4 The cartesian product A × A has 9 elements among which are found (-1, 0) and (0,1). Find the set A and the remaining elements of A × A.

5 Let A and B be two sets such that n (A) = 5 and n (B) = 2. If a, b, c, d, e are distinct and {a, 2), (b, 3), (c, 2), (d, 3), (e, 2) are in A × B. find A and B.

6 (i) If, $\left(\frac{a}{3}+, b-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$ find the values of a and b. (ii) If (x +1, 1) = (3, y - 2), find the values of x and y.

7 If the ordered pairs (x, - 1) and (5, y) belong to the set {(a, b) : b = 2a - 3}, find the values of x and y.

8 If a $\in$ {2,4,6,9} and b $\in$ {4,6,18,27}, then form the set of all ordered pairs (a, b) such that a divides b and a < b.

9 If A and B are two sets having 3 elements in common. If n(A) = 5, n(B) = 4, find n(A × B) and n[(A × B) $\cap$ (B × A)].

Let A and B be two sets. Show that the sets A × B and B × A have an element in common iff the sets A and B have an element in common.

10 Let A and B be two sets. Show that the sets A × B and B × A have an element in common iff the sets A and B have an element in common.

11 If A = {1, 2,3}, B = {4,5,6}, which of the following are relations from A to B? Give reasons in support of your answer.

(i) R_1 = {(1,4), (1,5), (1,6)} (ii) R_2 = {(1,5), (2,4), (3,6)}

(iii) R_3 = ((1,4), (1,5), (3,6), (2,6), (3,4)} (iv) R_4 = {(4,2), (2,6), (5,1), (2,4)}.

12 A relation R is defined on the set Z of integers as follows:

$$(x, y) \in R \Leftrightarrow x^2 + y^2 = 25$$

Express R and R^{-1} as the sets of ordered pairs and hence find their respective domains.

13 Let R be the relation on the set N of natural numbers defined by R = {(a, b): $a + 3b = 12$, $a \in N$, $b \in N$).

Find: (i) R (ii) Domain of R (iii) Range of R

14 Let A = {1,2,3,4,5,6}. Define a relation R on set A by R = {(x, y) : $y = x + 1$}

(i) Depict this relation using an arrow diagram.

(ii) Write down the domain, co-domain and range of P.

15 Let R be a relation on Q defined by

R = {(a,b) : $a,b \in Q$ and $a - b \in Z$} Show that:

(i) $(a, a) \in R$ for all $a \in Q$

(ii) $\{a, b) \in R \Rightarrow (b, a) \in R$

(iii) $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

16 Let R be a relation on N defined by

R = {(a,b) : $a,b \in N$ and $a = b^2$} Are the following true:

(i) $(a,a) \in R$ for all $a \in N$

(ii) $(a, b) \in R \Rightarrow (b, a) \in R$

(iii) $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$

(i) not true (ii) not true (iii) not true

17 Let R be the relation on the set Z of all integers defined by

$(x, y) \in R \Rightarrow x - y$ is divisible by n Prove that:

(i) $(x,x) \in R$ for all $x \in Z$

(ii) $(x, y) \in R \Rightarrow (y, x) \in R$ for all $x, y \in Z$

(iii) $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ for all $x, y, z \in R$.

18 If A = {1,2,3}, B = {4,5,6}, which of the following are relations from A to B ? Give reasons in support of your answer.

(i) {(1,6), (3,4), (5,2)}

(ii) {(1,5), (2,6), (3,4), (3,6)}

(iii) {(4,2), (4,3), (5,1)}

(iv) $A \times B$.

19 Let A be the set of first five natural numbers and let R be a relation on A defined as follows: $(x, y) \in R \Leftrightarrow x \leq y$

Express R and R^{-1} as sets of ordered pairs. Determine also (i) the domain of R^{-1} (ii) the range of R.

20 Write the following relations as the sets of ordered pairs:

(i) A relation R from the set {2,3,4,5,6} to the set {1,2,3} defined by $x = 2y$.

(ii) A relation R on the set {1,2,3,4,5,6,7} defined by $(x, y) \in R \Leftrightarrow x$ is relatively prime to y.

(iii) A relation R on the set {0,1,2,..., 10} defined by $2x + 3y = 12$.

(iv) A relation R from a set A = {5, 6, 7,8} to the set B = {10,12,15,16,18} defined by (x, y) 6 R $\Leftrightarrow$ x divides y.

21 Determine the domain and range of the following relations :

(i) $R = \{(a, b): a \in N, a < 5, b = 4\}$ (ii) $S = \{(a, b): b = |a - 1|, a \in Z \text{ and } |a| \leq 3\}$

22 Write the relation $R = \{(x, x^3): x$ is a prime number less than 10} in roster form.

23 Let R be a relation on N × N defined by

(a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c,d) $\in$ N × N Show that:

(i) (a, b) R (a, b) for all (a,b) $\in$ N × N

(ii) (a, b) R (c, d) $\Rightarrow$ (c,d) R (a, b) for all (a,b), (c,d) $\in$ N × N

(iii) (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f) for all (a, b), (c, d), (e, f) $\in$ N × N

24 If $R = \{(x,y) : x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation defined on the set Z of integers, then write domain of R.

25 If $R = \{(x, y): x, y \in W, 2x + y = 8\}$, then write the domain and range of R.

FUNCTIONS

26 Let f : R $\rightarrow$ R be given by $f(x) = x^2 + 3$. Find (a) {x : f(x) = 28} (b) the pre-images of 39 and 2 under f.

27 Let f : R $\rightarrow$ R be a function given by $f(x) = x^2 + 1$. Find :

(i) $f^{-1}\{-5\}$ (ii) $f^{-1}\{26\}$ (iii) $f^{-1}\{10, 37\}$

28 If f : R $\rightarrow$ R be defined as follows :

$$f(x) = \begin{cases} 1, & \text{if } x \in Q \\ -1, & \text{if } x \notin Q \end{cases}.$$

Find (a) $f^{(1/2)}$, $f(\pi)$, $f(\sqrt{2})$ (b) Range of f (c) pre-image of 1 and -1.

29 Let f : R $\rightarrow$ R be such that $f(x) = 2^x$. Determine :

(a) Range of f (b) {x : f(x) = 1} (c) whether f(x + y) = f(x) . f(y) holds

30 Let A = {-2, -1, 0, 1, 2} and f : A $\rightarrow$ Z be a function defined by $f(x) = x^2 - 2x - 3$. Find :

(a) range of f i.e. f(A) (b) pre-image of 6, -3 and 5

31 What is the fundamental difference between a relation and a function ? Is every relation a function ?

32 If a function $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 2 & , \quad x < 0; \\ 1 & , \quad x = 0; \\ 4x + 1 & , \quad x > 0 \end{cases}$$

Find : f(1), f(-1), f(0), f(2)

33 Let $f : R^+ \to R$, where R^+ is the set of all positive real numbers, be such that $f(x) = \log_e x$. Determine

(a) the image set of the domain of f (b) $\{x : f(x) = -2\}$

(c) whether f(xy) : f(x) + f(y) holds.

34 Let X = {1, 2, 3, 4} and Y = {1, 5, 9, 11, 15, 16}

Determine which of the following sets are functions from X to Y

(a) f_1 = {(1, 1), (2, 11), (3, 1), (4, 15)}

(b) f_2 = {(1, 1), (2, 7), (3, 5)}

(c) f_3 = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

35 If $f(x) = x + \frac{1}{x}$, prove that $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$.

36 If $f(x) = \frac{1}{2x+1}, x \neq -\frac{1}{2}$, then show that: $f\left(f(x)\right) = \frac{2x+1}{2x+3}$, provided that $x \neq -\frac{3}{2}$.

37 If f is a real function defined by $f(x) = \frac{x-1}{x+1}$, the prove that : $f(2x) = \frac{3f(x)+1}{f(x)+3}$.

38 If $f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ x, & \text{when } 0 \le x < 1 \\ \frac{1}{x}, & \text{when } x \ge 1 \end{cases}$: Find : (a) f(z/2), (b) f(-2), (c) f(1), (d) $f(\sqrt{3})$ and (e) $f(\sqrt{-3})$

39 If for non-zero x, $af(x) + b\, f\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then find f(x).

40 Find the domain of each of the following real valued functions :

(i) $f(x) = \frac{1}{x+2}$ (ii) $f(x) = \frac{x-1}{x-3}$ (iii) $f(x) = \frac{2x-3}{x^2-3x+2}$ (iv) $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$

41 Find the domain of each of the following functions :

(i) $f(x) = \sqrt{x-2}$ (ii) $f(x) = \frac{1}{\sqrt{1-x}}$ (iii) $f(x) = \sqrt{4-x^2}$

42 Find the domain and range of each of the following real valued functions :

(i) $f(x)=\dfrac{ax+b}{bx-a}$ (ii) $f(x)=\dfrac{ax-b}{cx-d}$ (iii) $f(x)=\sqrt{x-1}$ (iv) $f(x)=\sqrt{x-3}$

(v) $f(x)=\dfrac{x-2}{2-x}$ (vi) $f(x)=|x-1|$

43 Find the domain of each of the following function given by

(i) $f(x)=\dfrac{1}{\sqrt{x-|x|}}$ (ii) $f(x)=\dfrac{1}{\sqrt{x+|x|}}$ (iii) $f(x)=\dfrac{1}{\sqrt{x-[x]}}$ (iv) $f(x)=\dfrac{1}{\sqrt{x+[x]}}$

44 Find the domain of the function f(x) defined by $f(x)=\sqrt{4-x}+\dfrac{1}{\sqrt{x^2-1}}$.

45 Find the domain of definition of the function f(x) given by $f(x)=\log_4\left\{\log_5\left(\log_3(18x-x^2-77)\right)\right\}$

46 Find the domain of definition of the function f(x) given by $f(x)=\dfrac{1}{\log_{10}(1-x)}+\sqrt{x+2}$.

47 Find the domain and range of the function f(x) given by $f(x)=\dfrac{x-2}{3-x}$.

48 Find the domain of the real function f(x) defined by $f(x)=\sqrt{\dfrac{1-|x|}{2-|x|}}$

49 Find the domain and range of the function $f(x)=\dfrac{x^2-9}{x-3}$.

50 Find the domain and range of the real valued function f(x) given by $f(x)=\dfrac{4-x}{x-4}$.

51 Let $f=\left\{\left(x,\dfrac{x^2}{1+x^2}\right):x\in R\right\}$ be a function from R into R. Determine the range of f.

52 If f(x) = cos $[\pi^2]$ x + cos $[-\pi^2]$x, where [x] denotes the greatest integer less than or equal to x, then write the value of f(π).

53 Write the range of the function f(x) = sin [x], where $\dfrac{-\pi}{4}\le x\le\dfrac{\pi}{4}$.

54 Write the domain and range of $f(x)=\sqrt{x-[x]}$.

55 Write the range of the function f(x) = cos [x], where $\dfrac{-\pi}{2}<x<\dfrac{\pi}{2}$.

56 Write the range of the function f(x) = $e^{x-[x]}$, $x\in R$

57 Find the domain and range of the function $f=\left\{\left(x:\dfrac{1}{1-x^2}\right):x\in R, x\ne\pm1\right\}$

58 Find the domain and range of the function $f(x) = \frac{1}{2 - \sin 3x}$.

59 Let f and g be real functions defined by $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{4 - x^2}$. Then, find each of the following functions :

(i) $f + g$ (ii) $f - g$ (iii) fg (iv) $\frac{f}{g}$ (v) ff (vi) gg