CHAPTER – 2
Relation & Functions

2.1 Cartesian product of sets

Cartesian product of sets: Let A and B be any two non-empty sets. The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the cartesian product of the sets A and B and is denoted by $A \times B$.

Thus, $A \times B = [(a, b) : a \in A \text{ and } b \in B]$

If A = ¤ or B = ¤, then we define $A \times B$ = ¤.

Example **:** Let $A = \{a, b, c\}$ and $B = \{p, q\}$.

Then $A \times B = \{(a, p), (a, q), (b, p), (b, q), (c, p), (c, q)\}$

Also $B \times A = \{(p, a), (p, b), (p, c), (q, a), (q, b), (q, c)\}$

Important theorems on cartesian product of sets:

Theorem 1 : For any three sets A, B, C

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Theorem 2 : For any three sets A, B, C

$A \times (B - C) = (A \times B) - (A \times C)$

Theorem 3 : If A and B are any two non-empty sets, then

$A \times B = B \times A \Leftrightarrow A = B$

Theorem 4 : If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$

Theorem 5 : If $A \subseteq B$, then $A \times C \subseteq B \times C$ for any set C.

Theorem 6 : If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

Theorem 7 : For any sets A, B, C, D

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Theorem 8 : For any three sets A, B, C

(i) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

(ii) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$