2.2 Relations

Definition

Let A and B be two non-empty sets, then every subset of $A \times B$ defines a relation from A to B and every relation from A to B is a subset of $A \times B$.

Let $R \subseteq A \times B$ and $(a, b) \in R$. Then we say that a is related to b by the relation R and write it as $a\,R\,b$. If $(a, b) \in R$, we write it as $a\,R\,b$.

(1) **Total number of relations:** Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So, total number of subset of $A \times B$ is 2^{mn}. Since each subset of $A \times B$ defines relation from A to B, so total number of relations from A to B is 2^{mn}. Among these 2^{mn} relations the void relation ¤ and the universal relation $A \times B$ are trivial relations from A to B.

(2) **Domain and range of a relation:** Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R.

Thus, Dom $(R) = \{a : (a, b) \in R\}$ and Range $(R) = \{b : (a, b) \in R\}$.

Inverse relation

Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$

Clearly $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$. Also, Dom (R) = Range (R^{-1}) and Range (R) = Dom (R^{-1})

Example : Let $A = \{a, b, c\}$, $B = \{1, 2, 3\}$ and $R = \{(a, 1), (a, 3), (b, 3), (c, 3)\}$.

Then, (i) $R^{-1} = \{(1, a), (3, a), (3, b), (3, c)\}$

(ii) Dom $(R) = \{a, b, c\}$ = Range (R^{-1})

(iii) Range $(R) = \{1, 3\}$ = Dom (R^{-1})