2.3 Types of relations

(1) **Reflexive relation:** A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R is reflexive $\Leftrightarrow (a, a) \in R$ for all $a \in A$.

Example **:** Let $A = \{1, 2, 3\}$ and $R = \{(1, 1); (1, 3)\}$

Then R is not reflexive since $3 \in A$ but $(3, 3) \notin R$

A reflexive relation on A is not necessarily the identity relation on A.

The universal relation on a non-void set A is reflexive.

(2) **Symmetric relation:** A relation R on a set A is said to be a symmetric relation *iff* $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

i.e.,$aRb \Rightarrow bRa$ for all $a, b \in A$.

it should be noted that R is symmetric iff $R^{-1} = R$

The identity and the universal relations on a non-void set are symmetric relations.

A reflexive relation on a set A is not necessarily symmetric.

(3) **Anti-symmetric relation:** Let A be any set. A relation R on set A is said to be an anti-symmetric relation *iff* $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$ for all $a, b \in A$.

Thus, if $a \neq b$ then a may be related to b or b may be related to a, but never both.

(4) **Transitive relation:** Let A be any set. A relation R on set A is said to be a transitive relation *iff* $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$ *i.e.*, aRb and $bRc \Rightarrow aRc$ for all $a, b, c \in A$.

Transitivity fails only when there exists a, b, c such that $a R b$, $b R c$ but $a \not R c$.

Example **:** Consider the set $A = \{1, 2, 3\}$ and the relations

$R_1 = \{(1, 2), (1, 3)\}$; $R_2 = \{(1, 2)\}$; $R_3 = \{(1, 1)\}$;

$R_4 = \{(1, 2), (2, 1), (1, 1)\}$

Then R_1, R_2, R_3 are transitive while R_4 is not transitive since in $R_4, (2, 1) \in R_4; (1, 2) \in R_4$ but $(2, 2) \notin R_4$.

The identity and the universal relations on a non-void sets are transitive.

(5) **Identity relation:** Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A.

In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only. Every identity relation will be reflexive, symmetric and transitive.

Example **:** On the set = $\{1, 2, 3\}$, $R = \{(1, 1), (2, 2), (3, 3)\}$ is the identity relation on A .

It is interesting to note that every identity relation is reflexive but every reflexive relation need not be an identity relation.

(6) **Equivalence relation:** A relation R on a set A is said to be an equivalence relation on A *iff*

(i) It is reflexive *i.e.* $(a, a) \in R$ for all $a \in A$

(ii) It is symmetric *i.e.* $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$

(iii) It is transitive *i.e.* $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Congruence modulo (m) **:** Let m be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo m if $a - b$ is divisible by m and we write $a \equiv b$ (mod m).

Thus $a \equiv b$ (mod m) $\Leftrightarrow a - b$ is divisible by m. For example, $18 \equiv 3$ (mod 5) because 18 – 3 = 15 which is divisible by 5. Similarly, $3 \equiv 13$ (mod 2) because 3 – 13 = –10 which is divisible by 2. But 25 ≠ 2 (mod 4) because 4 is not a divisor of 25 – 3 = 22.

The relation "Congruence modulo m" is an equivalence relation.

FEW IMPORTANT RESULT:

- Equal sets are always equivalent but equivalent sets may need not be equal set.
- If A has n elements, then $P(A)$ has 2^n elements.
- The total number of subset of a finite set containing n elements is 2^n.
- If $A_1, A_2, \ldots\ldots, A_n$ is a finite family of sets, then their union is denoted by $\bigcup_{i=1}^{n} A_i$ or $A_1 \cup A_2 \cup A_3 \ldots\ldots \cup A_n$.
- If $A_1, A_2, A_3 \ldots\ldots, A_n$ is a finite family of sets, then their intersection is denoted by $\bigcap_{i=1}^{n} A_i$ or $A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n$.
- $R - Q$ is the set of all irrational numbers.
- Let A and B two non-empty sets having n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.
- The identity relation on a set A is an anti-symmetric relation.