2.4 Functions

If A and B are two non-empty sets, then a rule f which associated to each $x \in A$, a unique number $y \in B$, is called a function from A to B and we write, $f : A \to B$.

Some important definitions

(1) **Real numbers:** Real numbers are those which are either rational or irrational. The set of real numbers is denoted by R.

(2) Related quantities: When two quantities are such that the change in one is accompanied by the change in other, i.e., if the value of one quantity depends upon the other, then they are called related quantities.

(3) **Variable:** A variable is a symbol which can assume any value out of a given set of values.

(i) **Independent variable:** A variable which can take any arbitrary value, is called independent variable.

(ii) **Dependent variable:** A variable whose value depends upon the independent variable is called dependent variable.

(4) **Constant:** A constant is a symbol which does not change its value, *i.e.*, retains the same value throughout a set of mathematical operation. These are generally denoted by a, b, c etc. There are two types of constant, absolute constant and arbitrary constant.

(5) **Absolute value:** The absolute value of a number x, denoted by $|x|$, is a number that satisfies the conditions

$|x| = \begin{cases} -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$. We also define $|x|$ as follows,

$|x|$= maximum $\{x, -x\}$ or $|x| = \sqrt{x^2}$.

(6) **Fractional part:** We know that $x \geq [x]$. The difference between the number 'x' and it's integral value '$[x]$' is called the fractional part of x and is symbolically denoted as $\{x\}$. Thus, $\{x\} = x - [x]$ *e.g.*, if x = 4.92 then $[x]$ = 4 and $\{x\}$ = 0.92.

Fractional part of any number is always non-negative and less than one.

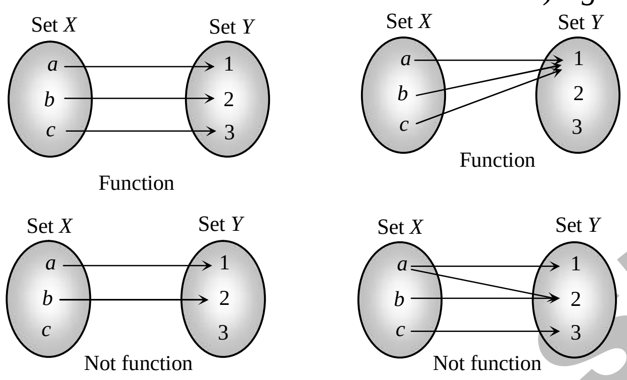
Definition of function

(1) Function can be easily defined with the help of the concept of mapping. Let X and Y be any two non-empty sets. "A function from X to Y is a rule or correspondence that assigns to each element of set X, one and only one element of set Y". Let the correspondence be 'f' then mathematically we write $f : X \to Y$ where $y = f(x), x \in X$ and $y \in Y$. We say that 'y' is the image of 'x' under f (or x is the pre image of y).

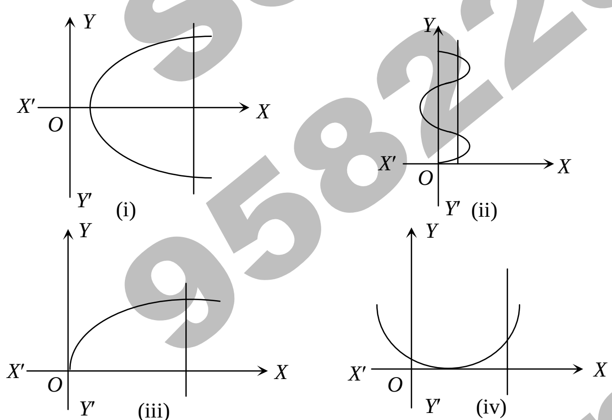
Two things should always be kept in mind:

(i) A mapping $f : X \to Y$ is said to be a function if each element in the set X has its image in set Y. It is also possible that there are few elements in set Y which are not the images of any element in set X.

(ii) Every element in set X should have one and only one image. That means it is impossible to have more than one image for a specific element in set X. Functions can not be multi-valued (A mapping that is multi-valued is called a relation from X and Y) *e.g.*



(2) Testing for a function by vertical line test: A relation $f : A \to B$ is a function or not it can be checked by a graph of the relation. If it is possible to draw a vertical line which cuts the given curve at more than one point then the given relation is not a function and when this vertical line means line parallel to Y-axis cuts the curve at only one point then it is a function. Figure (iii) and (iv) represents a function.



(3) **Number of functions:** Let X and Y be two finite sets having m and n elements respectively. Then each element of set X can be associated to any one of n elements of set Y. So, total number of functions from set X to set Y is n^m.

(4) Value of the function: If $y = f(x)$ is a function then to find its values at some value of x, say $x = a$, we directly substitute x = a in its given rule $f(x)$ and it is denoted by $f(a)$.

e.g. If $f(x) = x^2 + 1$, then $f(1) = 1^2 + 1 = 2$, $f(2) = 2^2 + 1 = 5$, $f(0) = 0^2 + 1 = 1$ *etc.*

Sonu Sir 9582289230