Chapter - 3

ASSIGNMENT

1 Find the degree measure corresponding to the following radian measures :

(i) $\left(\frac{1}{4}\right)^c$ (ii) -2^c (iii) 6^c (iv) $\left(\frac{11}{16}\right)^c$

2 Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°.

3 In a circle of diameter 40 cm the length of a chord is 20 cm. Find the length of minor arc corresponding to the chord.

45 The angles of a triangle are in A.P. The number of degrees in the least is to the number of radians in the greatest as 60 : π. Find the angles in degrees.

6 A circular wire of radius 7.5 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 120 cm. Find the degrees the angle which is subtended at the centre of the hoop.

7 If the angular diameter of the moon be 30', how far from the eye a con of diameter 2.2 cm be kept of hide the moon ?

8 The angle in one regular polygon is to that in another as 3 : 2 and the number of sides in first is twice that in the second. Determine the number of sides of two polygons.

9 Find the diameter of the sun in km supposing that it subtends an angles of 32' at the eye of an observer. Given that the distance of the sun is 91×10^6 km.

10 Find the values of cos θ and tan θ if $\sin\theta = -\frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$.

11 Find all other trigonometric ratios of $\sin\theta = -\frac{2\sqrt{6}}{5}$ and θ lies in quadrant III.

12 If $\sec\theta = \sqrt{2}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find the value of $\frac{1+\tan\theta+\operatorname{cosec}\theta}{1+\cot\theta-\operatorname{cosec}\theta}$.

13 Prove that :

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \begin{cases} \sec\theta - \tan\theta, & \text{if } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\ -\sec\theta + \tan\theta, & \text{if } \frac{\pi}{2} < \theta < \frac{3\pi}{2} \end{cases}$$

14 If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then write the value of $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$.

15 If $3\sin\theta + 5\cos\theta = 5$, then write the value of $5\sin\theta - 3\cos\theta$.

16 Write the value of $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$.

17 If $\sin x + \sin^2 x = 1$, then write the value of $\cos^8 x + 2\cos^6 x + \cos^4 x$.

18 If $\sin x + \text{cosec } x = 2$, then write the value of $\sin^n x + \text{cosec}^n x$.

19 Prove that :

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \begin{cases} \text{cosec}\theta + \cot\theta, & \text{if } 0 < \theta < \pi \\ -\text{cosec}\theta - \cot\theta, & \text{if } \pi < \theta < 2\pi \end{cases}$$

20 If $\sin\theta = \frac{3}{5}$, $\tan\phi = \frac{1}{2}$ and $\frac{\pi}{2} < \theta < \pi < \phi < \frac{3\pi}{2}$, find the value of $8\tan\theta - \sqrt{5}\sec\phi$.

21 Prove that : $\cos 510° \cos 330° + \sin 390° \cos 120° = -1$.

22 Prove that : $\sin(-420°)(\cos 390°) + \cos(-660°)(\sin 330°) = -1$

23 If A, B, C, D are angles of a cyclic quadrilateral, prove that
$\cos A + \cos B + \cos C + \cos D = 0$.

24 Prove that :

(i) $\tan 225° \cot 405° + \tan 765° \cot 675° = 0$

(ii) $\cos 24° + \cos 55° + \cos 125° + \cos 204° + \cos 300° = \frac{1}{2}$

(iii) $\tan\frac{11\pi}{3} - 2\sin\frac{4\pi}{6} - \frac{3}{4}\text{cosec}^2\frac{\pi}{4} + 4\cos^2\frac{17\pi}{6} = \frac{3-4\sqrt{3}}{2}$

(iv) $3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4} = 1$

25 If A, B, C, D be the angles of a cyclic quadrilateral, taken in order, prove that
$\cos(180° - A) + \cos(180° + B) + \cos(180° + C) - \sin(90° + D) = 0$

26 Find x from the following equations :

(i) $\text{cosec}(90° + \theta) + x\cos\theta\cot(90° + \theta) = \sin(90° + \theta)$

(ii) x cot (90° + θ) + tan (90° + θ) sin θ + cosec (90° + θ) = 0

27 If $\cot\alpha = \frac{1}{2}$, $\sec\beta = \frac{-5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$. Find the value of tan(α + β).

State the quadrant in which α + β terminates.

28 Find the sign of the expression sin 100° + cos 100°.

29 Prove that tan 75° + cot 75° = 4

30 Prove that : $\frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A} + \frac{\sin(A-B)}{\cos A\cos B} = 0$

31 If cos (α - β) + cos (β - γ) + cos (γ - α) = $-\frac{3}{2}$, prove that :

cos α + cos β + cos γ = sin α + sin β + sin γ = 0

32 Prove that :

(i) tan 3A tan 2A tan A = tan 3A – tan 2A – tan A

(ii) cot A cot 2A – cot 2A cot 3A – cot 3A cot A = 1

33 If α and β are acute angles such that $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, prove that $\alpha + \beta = \frac{\pi}{4}$.

34 Prove that : $\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right) = \frac{1}{\sqrt{2}}\sin A$

35 If cos (α + β) = $\frac{4}{5}$, sin (α - β) = $\frac{5}{13}$ and α, β lie between 0 and $\frac{\pi}{4}$, prove that $\tan 2\alpha = \frac{56}{33}$.

36 If tan (π cos θ) = cot (π sin θ), prove that $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$

37 If sin α + sin β = a and cos α + cos β = b, show that

(i) $\cos(\alpha+\beta) = \frac{b^2 - a^2}{b^2 + a^2}$ (ii) $\sin(\alpha+\beta) = \frac{2ab}{a^2 + b^2}$

38 Prove that : $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

39 Prove that : cos 20° cos 40° cos 60° cos 80° = $\frac{1}{16}$

40 Prove that : sin 10° sin 30° sin 50° sin 70° = $\frac{1}{16}$.

41 Prove that : tan 20° tan 40° tan 80° = tan 60°

42 Prove that sin A sin (60° - A) sin (60° + A) = $\frac{1}{4}$ sin 3A

43 If $\alpha + \beta = 90°$, find the maximum and minimum values of sin α sin β.

44 Show that :

(i) sin A sin (B – C) + sin B sin(C – A) + sin C sin(A – B) = 0

(ii) sin(B – C) cos (A – D) + sin (C – A) cos (B – D) + sin (A – B) cos (C – D) = 0

45 Prove that : $\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=4\cos\frac{\alpha+\beta}{2}\cos\frac{\beta+\gamma}{2}\cos\frac{\gamma+\alpha}{2}$

46 Prove that : $\frac{\sin A+\sin 3A+\sin 5A+\sin 7A}{\cos A+\cos 3A+\cos 5A+\cos 7A}=\tan 4A$

47 Prove that :

$$\frac{\cos 2A\cos 3A-\cos 2A\cos 7A+\cos A\cos 10A}{\sin 4A\sin 3A-\sin 2A\sin 5A+\sin 4A\sin 7A}=\cot 6A\cot 5A$$

48 Prove that : $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n=\begin{cases}2\cot^n\left(\frac{A-B}{2}\right), & \text{if n is even}\\ 0, & \text{if n is odd}\end{cases}$

49 Prove that : $\frac{\sin(\theta+\phi)-2\sin\theta+\sin(\theta-\phi)}{\cos(\theta+\phi)-2\cos\theta+\cos(\theta-\phi)}=\tan\theta$

50 Prove that :

(i) $\sin\alpha+\sin\beta+\sin\gamma-\sin(\alpha+\beta+\gamma)=4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\gamma+\alpha}{2}\right)$

(ii) cos (A + B + C) + cos (A – B + C) + cos (A + B – C) + cos (-A + B + C) = 4 cos A cos B cos C

51 Show that : $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

52 Prove that : $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

53 If $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$, then write the value of $\sqrt{\frac{1+\cos 2\theta}{2}}$.

54 If $\pi < \theta < \frac{3\pi}{2}$, then write the value of $\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$.

55 Prove that : $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right) = \frac{1}{8}$

56 Prove that :

(i) $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$ (ii) $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

57 Show that $\sqrt{3}\,\text{cosec}\,20° - \sec 20° = 4$

58 If $\sin A = \frac{3}{5}$, where $0° < A < 90°$, find the values of sin 2A, cos 2A, tan 2A and sin 4A

59 Find the value of $\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$

60 Prove that :

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

61 Prove that : $\tan\left(\frac{\pi}{4}+\theta\right) + \tan\left(\frac{\pi}{4}-\theta\right) = 2\sec 2\theta$

62 If $\sin\alpha = \frac{4}{5}$ and $\cos\beta = \frac{5}{13}$, prove that $\cos\frac{\alpha-\beta}{2} = \frac{8}{\sqrt{65}}$

63 Prove that : $\cos A \cos(60 - A) \cos(60 + A) = \frac{1}{4}\cos 3A$

64 If $\cos\alpha + \cos\beta + \cos\gamma = 0$, then prove that

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 12\cos\alpha\cos\beta\cos\gamma$

65 Prove that : $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

66 Find the general solutions of the following equations:

(i) $\cos 3\theta = 0$ (ii) $\cos\left(\frac{3}{2}\theta\right) = 0$ (iii) $\cos 2\ 3\theta = 0$

67 Solve the following trigonometric equations :

(i) $\tan\theta = \frac{1}{\sqrt{3}}$ (ii) $\tan 2\theta = \sqrt{3}$ (iii) $\tan 3\theta = -1$

68 Solve the equation : $\cos\theta + \cos 3\theta - 2\cos 2\theta = 0$

69 Solve the equation : $\sin m\theta + \sin n\theta = 0$.

70 Solve the following equations :

(i) $2\cos 2\theta + 3\sin\theta = 0$ (ii) $\cot^2\theta + \frac{3}{\sin\theta} + 3 = 0$

(iii) $2\tan\theta - \cot\theta = -1$ (iv) $4\cos\theta - 3\sec\theta = \tan\theta$

(v) $\tan 2\theta + (1 - \sqrt{3})\tan\theta - \sqrt{3} = 0$ (vi) $\sec 2\ 2x = 1 - \tan 2x$

71 Solve the following equations : $\tan\theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$

72 Solve : $7\cos 2\theta + 3\sin 2\theta = 4$

73 Solve : $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$

74 Solve : $\cot\theta + \operatorname{cosec}\theta = \sqrt{3}$

75 Solve the following equations :

(i) $\sin\theta + \cos\theta = \sqrt{2}$ (ii) $\sqrt{3}\cos\theta + \sin\theta = 1$

76 Write the set of values of a for which the equation $\sqrt{3}\sin x - \cos x = a$ has no solution.

77 Write the values of x in $[0, \pi]$ for which $\sin 2x$, $\frac{1}{2}$ and $\cos 2x$ are in AP.

78 Write the number of values of θ in $[0, 2\pi]$ that satisfy the equation $\sin^2\theta - \cos\theta = \frac{1}{4}$.