3.2 System of measurement of angles

There are three system for measuring angles

(1) **Sexagesimal or English system:** Therefore,

1 right angle = 90 degree $(=90^o)$

$1^o = 60$ minutes $(=60')$

$1' = 60$ second $(=60'')$

(2) **Centesimal or French system:** Therefore,

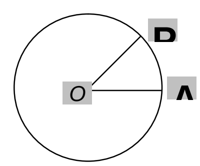
1 right angle = 100 grades $(=100^g)$

1 grade = 100 minutes $(=100')$

1 minute = 100 seconds $(=100'')$

(3) **Circular system:** The measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

Consider a circle of radius *r* having centre at *O*. Let *A* be a point on the circle. Now cut off an arc *AP* whose length is equal to the radius *r* of the circle. Then by the definition the measure of $\angle AOP$ is 1 radian $(=1^c)$.



Relation between three systems of measurement of an angle

Let *D* be the number of degrees, *R* be the number of radians and *G* be the number of grades in an angle −, then

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{®}$$

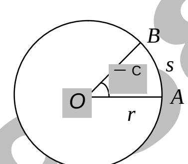
This is the required relation between the three systems of measurement of an angle.

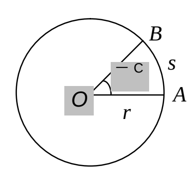
Therefore, one radian $= \frac{180^o}{®} \Rightarrow ®$ radians $= 180^o$

i.e., 1 radian = $57°17'44.8'' \approx 57^o17'45''$.

Relation between an arc and an angle

If *s* is the length of an arc of a circle of radius *r*, then the angle − (in radians) subtended by this arc at the centre of the circle is given by $− = \frac{s}{r}$ or $s = r\theta$.





i.e., Arc = radius × angle in radians

Sectorial area: Let *OAB* be a sector having central angle $^{-c}$ and radius *r*. Then area of the sector *OAB* is given by $\frac{1}{2}r^2\theta$.

Domain and range of a trigonometrical function

If $f : X \to Y$ is a function, defined on the set *X*, then the **domain** of the function *f*, written as Domf is the set of all independent variables *x*, for which the image $f(x)$ is well defined element of *Y*, called the co-domain of *f*.

Range of $f : X \to Y$ is the set of all images $f(x)$ which belongs to *Y*, *i.e.*, Range $f =$ $\{f(x) \in Y : x \in X\} \subseteq Y$.

The domain and range of trigonometrical functions are tabulated as follows :

Table : 10.1

Trigonometrical Function	**Domain**	**Range**
$\sin x$	R	$-1 \le \sin x \le 1$
$\cos x$	R	$-1 \le \cos x \le 1$
$\tan x$	$R - \left\{(2n+1)\frac{\circledR}{2}, n \in I\right\}$	**R**
$\text{cosec } x$	$R - \{n\circledR, n \in I\}$	$R - \{x : -1 < x < 1\}$
$\sec x$	$R - \left\{(2n+1)\frac{\circledR}{2}, n \in I\right\}$	$R - \{x : -1 < x < 1\}$
$\cot x$	$R - \{n\circledR, n \in I\}$	R