

3.3 Trigonometrical ratios or functions

In the right angled triangle OMP , we have base = $OM = x$, perpendicular = $PM = y$ and hypotenues = $OP = r$. We define the following trigonometric ratio which are also known as trigonometric function.

$$\sin q = \frac{\text{Perpendicular}}{\text{Hypotenues}} = \frac{y}{r}$$

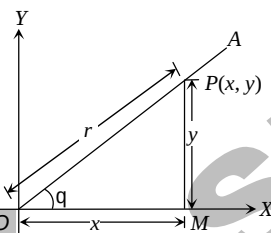
$$\cos q = \frac{\text{Base}}{\text{Hypotenues}} = \frac{x}{r}$$

$$\tan q = \frac{\text{Perpendicular}}{\text{Base}} = \frac{y}{x}$$

$$\cot q = \frac{\text{Base}}{\text{Perpendicular}} = \frac{x}{y}$$

$$\sec q = \frac{\text{Hypotenues}}{\text{Base}} = \frac{r}{x}$$

$$\operatorname{cosec} q = \frac{\text{Hypotenues}}{\text{Perpendicular}} = \frac{r}{y}$$



(1) Relation between trigonometric ratios (functions)

$$(i) \sin q \cdot \operatorname{cosec} q = 1$$

$$(ii) \tan q \cdot \cot q = 1$$

$$(iii) \cos q \cdot \sec q = 1$$

$$(iv) \tan q = \frac{\sin q}{\cos q} \quad (v) \cot q = \frac{\cos q}{\sin q}$$

(2) Fundamental trigonometric identities

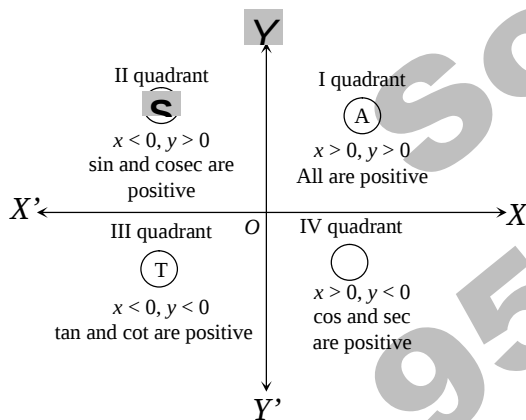
$$(i) \sin^2 q + \cos^2 q = 1$$

$$(ii) 1 + \tan^2 q = \sec^2 q$$

$$(iii) 1 + \cot^2 q = \operatorname{cosec}^2 q$$

(3) **Sign of trigonometrical ratios or functions:** Their signs depends on the quadrant in which the terminal side of the angle lies.

In brief: A crude aid to memorise the signs of trigonometrical ratio in different quadrant. "Add Sugar To Coffee".

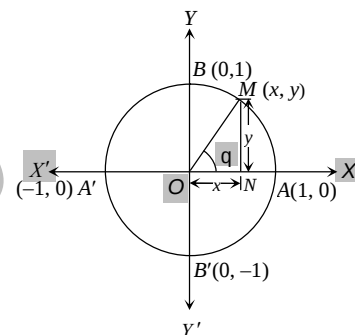


Algorithm : First determine the sign of the trigonometric function.

If q is measured from $X'OX$ i.e., $\{(p \pm q, 2p - q)\}$ then retain the original name of the function.

If q is measured from $Y'OY$ i.e., $\{\frac{p}{2} \pm q, \frac{3p}{2} \pm q\}$, then change sine to cosine, cosine to sine, tangent to cotangent, cot to tan, sec to cosec and cosec to sec.

(4) Variations in values of trigonometric functions in different quadrants: Let $X'OX$ and $Y'OY$ be the coordinate axes. Draw a circle with centre at origin O and radius unity.



Let $M(x, y)$ be a point on the circle such that $\angle AOM = q$ then $x = \cos q$ and $y = \sin q$; $-1 \leq \cos q \leq 1$ and $-1 \leq \sin q \leq 1$ for all values of q .

Table : 10.2

II-Quadrant (S)

$\sin q \rightarrow$ decreases from 1 to 0
 $\cos q \rightarrow$ decreases from 0 to -1
 $\tan q \rightarrow$ increases from $-\infty$ to 0
 $\cot q \rightarrow$ decreases from 0 to $-\infty$
 $\sec q \rightarrow$ increases from $-\infty$ to -1
 $\csc q \rightarrow$ increases from 1 to ∞

III-Quadrant (T)

$\sin q \rightarrow$ decreases from 0 to -1
 $\cos q \rightarrow$ increases from -1 to 0
 $\tan q \rightarrow$ increases from 0 to ∞
 $\cot q \rightarrow$ decreases from ∞ to 0
 $\sec q \rightarrow$ decreases from -1 to $-\infty$
 $\csc q \rightarrow$ increases from $-\infty$ to -1

I-Quadrant (A)

$\sin q \rightarrow$ increases from 0 to 1
 $\cos q \rightarrow$ decreases from 1 to 0
 $\tan q \rightarrow$ increases from 0 to ∞
 $\cot q \rightarrow$ decreases from ∞ to 0
 $\sec q \rightarrow$ increases from 1 to ∞
 $\csc q \rightarrow$ decreases from ∞ to 1

IV-Quadrant (C)

$\sin q \rightarrow$ increases from -1 to 0
 $\cos q \rightarrow$ increases from 0 to 1
 $\tan q \rightarrow$ increases from $-\infty$ to 0
 $\cot q \rightarrow$ decreases from 0 to $-\infty$
 $\sec q \rightarrow$ decreases from ∞ to 1
 $\csc q \rightarrow$ decreases from -1 to $-\infty$

Trigonometrical ratios of allied angles

Two angles are said to be allied when their sum or difference is either zero or a multiple of 90° .

Table : 10.3

Allied angles Trigo. Ratio	$\sin q$	$\cos q$	$\tan q$
$\downarrow$ $(-q)$	$-\sin q$	$\cos q$	$-\tan q$
$(90 - q)$ or $\left(\frac{\pi}{2} - \theta\right)$	$\cos q$	$\sin q$	$\cot q$
$(90 + q)$ or $\left(\frac{\pi}{2} + \theta\right)$	$-\cos q$	$-\sin q$	$-\cot q$
$(180 - q)$ or $(p - q)$	$\sin q$	$-\cos q$	$-\tan q$
$(180 + q)$ or $(p + q)$	$-\sin q$	$-\cos q$	$\tan q$
$(270 - q)$ or $\left(\frac{3\pi}{2} - \theta\right)$	$-\cos q$	$-\sin q$	$\cot q$
$(270 + q)$ or $\left(\frac{3\pi}{2} + \theta\right)$	$\cos q$	$\sin q$	$-\cot q$
$(360 - q)$ or $(2p - q)$	$-\sin q$	$\cos q$	$-\tan q$

Trigonometrical ratios for various angles

q	0	$p/6$	$p/4$	$p/3$	$p/2$	p	$3p/2$	$2p$
$\sin q$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	-1	0
$\cos q$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1	0	1
$\tan q$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	∞	0

Trigonometrical ratios for some special angles

q	$7\frac{1^\circ}{2}$	15°	$22\frac{1^\circ}{2}$	18°	36°
$\sin q$	$\frac{\sqrt{4-\sqrt{2}-\sqrt{6}}}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$
$\cos q$	$\frac{\sqrt{4+\sqrt{2}+\sqrt{6}}}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{\sqrt{5}+1}{4}$
$\tan q$	$\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{2}-1)}$	$2-\sqrt{3}$	$\sqrt{2}-1$	$\frac{\sqrt{25-10\sqrt{15}}}{5}$	$\sqrt{5-2\sqrt{5}}$

Trigonometrical ratios in terms of each other

	$\sin q$	$\cos q$	$\tan q$	$\cot q$	$\sec q$	$\operatorname{cosec} q$
$\sin q$	$\sin q$	$\sqrt{1-\cos^2 q}$	$\frac{\tan q}{\sqrt{1+\tan^2 q}}$	$\frac{1}{\sqrt{1+\cot^2 q}}$	$\frac{\sqrt{\sec^2 q - 1}}{\sec q}$	$\frac{1}{\operatorname{cosec} q}$
$\cos q$	$\sqrt{1-\sin^2 q}$	$\cos q$	$\frac{1}{\sqrt{1+\tan^2 q}}$	$\frac{\cot q}{\sqrt{1+\cot^2 q}}$	$\frac{1}{\sec q}$	$\frac{\sqrt{\operatorname{cosec}^2 q - 1}}{\operatorname{cosec} q}$
$\tan q$	$\frac{\sin q}{\sqrt{1-\sin^2 q}}$	$\frac{\sqrt{1-\cos^2 q}}{\cos q}$	$\tan q$	$\frac{1}{\cot q}$	$\sqrt{\sec^2 q - 1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2 q - 1}}$
$\cot q$	$\frac{\sqrt{1-\sin^2 q}}{\sin q}$	$\frac{\cos q}{\sqrt{1-\cos^2 q}}$	$\frac{1}{\tan q}$	$\cot q$	$\frac{1}{\sqrt{\sec^2 q - 1}}$	$\sqrt{\operatorname{cosec}^2 q - 1}$
$\sec q$	$\frac{1}{\sqrt{1-\sin^2 q}}$	$\frac{1}{\cos q}$	$\sqrt{1+\tan^2 q}$	$\frac{\sqrt{1+\cot^2 q}}{\cot q}$	$\sec q$	$\frac{\operatorname{cosec} q}{\sqrt{\operatorname{cosec}^2 q - 1}}$
$\operatorname{cosec} q$	$\frac{1}{\sin q}$	$\frac{1}{\sqrt{1-\cos^2 q}}$	$\frac{\sqrt{1+\tan^2 q}}{\tan q}$	$\sqrt{1+\cot^2 q}$	$\frac{\sec q}{\sqrt{\sec^2 q - 1}}$	$\operatorname{cosec} q$