3.5 Trigonometrical Equations and Inequations

Definition

An equation involving one or more trigonometrical ratio of an unknown angle is called a trigonometrical equation

i.e., $\sin x + \cos 2x = 1$, $(1 - \tan\theta)(1 + \sin 2\theta) = 1 + \tan\theta$,

$\left| \sec\left(\theta + \frac{\pi}{4}\right) \right| = 2$ etc.

A trigonometric equation is different from a trigonometrical identities. An identity is satisfied for every value of the unknown angle *e.g.,* $\cos^2 x = 1 - \sin^2 x$ is true $\forall x \in R$, while a trigonometric equation is satisfied for some particular values of the unknown angle.

(1) **Roots of trigonometrical equation:** The value of unknown angle (a variable quantity) which satisfies the given equation is called the root of an equation, *e.g.*, $\cos\theta = \frac{1}{2}$, the root is $\theta = 60^o$ or $\theta = 300^o$ because the equation is satisfied if we put $\theta = 60^o$ or $\theta = 300^o$.

(2) **Solution of trigonometrical equations:** A value of the unknown angle which satisfies the trigonometrical equation is called its solution.

Since all trigonometrical ratios are periodic in nature, generally a trigonometrical equation has more than one solution or an infinite number of solutions. There are basically three types of solutions:

(i) **Particular solution:** A specific value of unknown angle satisfying the equation.

(ii) **Principal solution:** Smallest numerical value of the unknown angle satisfying the equation (Numerically smallest particular solution).

(iii) **General solution:** Complete set of values of the unknown angle satisfying the equation. It contains all particular solutions as well as principal solutions.

Trigonometrical equations with their general solution

Trigonometrical equation	General solution
$\sin\theta = 0$	$\theta = n\pi$
$\cos\theta = 0$	$\theta = n\pi + \pi/2$
$\tan\theta = 0$	$\theta = n\pi$
$\sin\theta = 1$	$\theta = 2n\pi + \pi/2$
$\cos\theta = 1$	$\theta = 2n\pi$
$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n \alpha$

$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha$
$\tan\theta = \tan\alpha$	$\theta = n\pi \pm \alpha$
$\sin^2\theta = \sin^2\alpha$	$\theta = n\pi \pm \alpha$
$\tan^2\theta = \tan^2\alpha$	$\theta = n\pi \pm \alpha$
$\cos^2\theta = \cos^2\alpha$	$\theta = n\pi \pm \alpha$
$\left.\begin{array}{l}\sin\theta = \sin\alpha \\ \cos\theta = \cos\alpha\end{array}\right| *$	$\theta = 2n\pi + \alpha$
$\left.\begin{array}{l}\sin\theta = \sin\alpha \\ \tan\theta = \tan\alpha\end{array}\right| *$	$\theta = 2n\pi + \alpha$
$\left.\begin{array}{l}\tan\theta = \tan\alpha \\ \cos\theta = \cos\alpha\end{array}\right| *$	$\theta = 2n\pi + \alpha$