3.6 General solution of the form a cos¯ + b sin¯ = c

In $a\cos^{-} + b\sin^{-} = c,$ put $a = r\cos\sim$ and $b = r\sin\sim$ where $r = \sqrt{a^2 + b^2}$ and $|\ c| \le \sqrt{a^2 + b^2}$

Then, $r(\cos\sim\cos^{-} + \sin\sim\sin^{-}) = c$

$\Rightarrow \cos(^{-} - \sim) = \dfrac{c}{\sqrt{a^2 + b^2}} = \cos$, (say) .....(i)

$\Rightarrow {}^{-} - \sim = 2n® \pm \quad \Rightarrow {}^{-} = 2n® \pm \quad + \sim,$ where $\tan\sim = \dfrac{b}{a}$ is the general solution.

Alternatively, putting $a = r\sin\sim$ and $b = r\cos\sim$,

where $r = \sqrt{a^2 + b^2} \Rightarrow \sin(^{-} + \sim) = \dfrac{c}{\sqrt{a^2 + b^2}} = \sin¥$, (say)

$\Rightarrow {}^{-} + \sim = n® + (-1)^n¥ \Rightarrow {}^{-} = n® + (-1)^n¥ - \sim,$

where $\tan\sim = \dfrac{a}{b}$ is the general solution.

$(-\sqrt{a^2 + b^2}) \le a\cos^{-} + b\sin^{-} \le (\sqrt{a^2 + b^2})$

The general solution of $a\cos x + b\sin x = c$ is

$x = 2n® + \tan^{-1}\left(\dfrac{b}{a}\right) \pm \cos^{-1}\left(\dfrac{c}{\sqrt{a^2 + b^2}}\right).$