

3.6 General solution of the form $a \cos q + b \sin q = c$

In $a \cos q + b \sin q = c$, put $a = r \cos a$ and $b = r \sin a$ where $r = \sqrt{a^2 + b^2}$ and $|c| \leq \sqrt{a^2 + b^2}$

Then, $r(\cos a \cos q + \sin a \sin q) = c$

$$\Rightarrow \cos(q - a) = \frac{c}{\sqrt{a^2 + b^2}} = \cos b, \text{ (say)} \quad \dots(i)$$

$\Rightarrow q - a = 2np \pm b \Rightarrow q = 2np \pm b + a$, where $\tan a = \frac{b}{a}$ is the general solution.

Alternatively, putting $a = r \sin a$ and $b = r \cos a$,

where $r = \sqrt{a^2 + b^2} \Rightarrow \sin(q + a) = \frac{c}{\sqrt{a^2 + b^2}} = \sin g$, (say)

$$\Rightarrow q + a = np + (-1)^n g \Rightarrow q = np + (-1)^n g - a,$$

where $\tan a = \frac{a}{b}$ is the general solution.

$$(-\sqrt{a^2 + b^2}) \leq a \cos q + b \sin q \leq (\sqrt{a^2 + b^2})$$

The general solution of $a \cos x + b \sin x = c$ is

$$x = 2np + \tan^{-1}\left(\frac{b}{a}\right) \pm \cos^{-1}\left(\frac{c}{\sqrt{a^2 + b^2}}\right).$$

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