CHAPTER – 4
PMI

4.1 Introduction

First principle of Mathematical induction

The proof of proposition by mathematical induction consists of the following three steps :

Step I : (Verification step) **:** Actual verification of the proposition for the starting value "i".

Step II : (Induction step) **:** Assuming the proposition to be true for "k", $k \geq i$ and proving that it is true for the value $(k + 1)$ which is next higher integer.

Step III : (Generalization step) **:** To combine the above two steps. Let $p(n)$ be a statement involving the natural number n such that (i) $p(1)$ is true *i.e.* $p(n)$ is true for $n = 1$.

(ii) $p(m + 1)$ is true, whenever $p(m)$ is true *i.e.* $p(m)$ is true

$\Rightarrow p(m + 1)$ is true.

Then $p(n)$ is true for all natural numbers n.

Second principle of Mathematical induction

The proof of proposition by mathematical induction consists of following steps :

Step I : (Verification step) : Actual verification of the proposition for the starting value i and $(i + 1)$.

Step II : (Induction step) : Assuming the proposition to be true for $k - 1$ and k and then proving that it is true for the value $k + 1$; $k \geq i + 1$.

Step III : (Generalization step) : Combining the above two steps. Let $p(n)$ be a statement involving the natural number n such that (i) $p(1)$ is true *i.e.* $p(n)$ is true for $n = 1$ and

(ii) $p(m + 1)$ is true, whenever $p(n)$ is true for all n, where $i \leq n \leq m$.

Then $p(n)$ is true for all natural numbers.

For $a \neq b$, The expression $a^n - b^n$ is divisible by

(a) $a + b$, if n is even.

(b) $a - b$, if n is odd or even.

Divisibility problems

To show that an expression is divisible by an integer

(i) If a, p, n, r are positive integers, then first of all we write $a^{pn+r} = a^{pn} . a^{r} = (a^{p})^{n} . a^{r}$.

(ii) If we have to show that the given expression is divisible by c.

Then express, $a^{p} = [1 + (a^{p} - 1)]$, if some power of $(a^{p} - 1)$ has c as a factor. $a^{p} = [2 + (a^{p} - 2)]$, if some power of $(a^{p} - 2)$ has c as a factor.

$a^{p} = [k + (a^{p} - k)]$, if some power of $(a^{p} - k)$ has c as a factor.