Chapter - 5

ASSIGNMENT

1 Show that:

(i) $\left\{i^{19}+\left(\frac{1}{i}\right)^{25}\right\}^2=-4$ (ii) $\left\{i^{17}-\left(\frac{1}{i}\right)^{34}\right\}^2=2i$ (iii) $\left\{i^{18}+\left(\frac{1}{i}\right)^{24}\right\}^3=0$

(iv) $i^n+i^{n+1}+i^{n+2}+i^{n+3}=0,$ for all $n\in N.$

2 Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number.

3 Find the values of the following expressions:

(i) $i^{49} + i^{68} + i^{89} + i^{110}$ (ii) $i^{30} + i^{80} + i^{120}$ (iii) $i + i^2 + i^3 + i^4$

(iv) $i^5 + i^{10} + i^{15}$ (v) $\dfrac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$ (vi) $1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}$

4 A student writes the formulae $\sqrt{ab}=\sqrt{a}.\sqrt{b}$. Then he substitutes a = -1 and b = -1 and finds 1 = -1. Explain where is he wrong ?

5 Express each of the following in the from a + ib:

(i) $\left(\frac{1}{5}+\frac{2}{5}i\right)-\left(4+\frac{5}{2}i\right)$ (ii) $\left\{\left(\frac{1}{3}+\frac{7}{3}i\right)+\left(4+\frac{1}{3}i\right)\right\}-\left(-\frac{4}{3}+i\right)$

6 Express each one of the following in the standard from a +ib:

(i) $\dfrac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ (ii) $\left(\dfrac{1}{1-2i}+\dfrac{3}{1+i}\right)\left(\dfrac{3+4i}{2-4i}\right)$

(iii) $\dfrac{1}{1-\cos\theta+2i\sin\theta}$ (iv) $\dfrac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$

7 Express $(1-2i)^{-3}$ in the standard from a + ib.

8 Find the multiplicative inverse of the following complex number:

(i) $3+2i$ (ii) $(2+\sqrt{3}i)^2$

9 Find the least positive value n, if $\left(\frac{1+i}{1-i}\right)^n = 1$.

10 If $z_{1,}z_2$ are 1 – i , - 2 + 4i, respectively, find Im $\left(\frac{z_1 z_2}{\overline{z_1}}\right)$

11 Find the real values of x and y, if

(i) (3x - 7) + 2iy = - 5y + (5 + x)i (ii) (1 - i)x + (1 + i)y = 1 - 3i

(iii) (x + iy)(2 - 3i) = 4 + i (iv) $\frac{x-1}{3+i} + \frac{y-1}{3-i} = i$

12 If a + ib = $\frac{c+i}{c-i}$ where c is real, prove that:

$$a^2 + b^2 = 1 \text{and} \frac{b}{a} = \frac{2c}{c^2-1}$$

13 If $(x+iy)^{1/3}$ = a + ib, x , y, a b $\in$ R. Show that $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$.

14 Find the real values of x and y for which the complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other.

15 If $\frac{a+ib}{c+id} = x + iy$, prove $\frac{a-ib}{c-id} = x - iy$ and $\frac{a^2+b^2}{c^2+d^2} = x^2 + y^2$.

16 If a + ib $= \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

17 If (1 + i)(1 + 2i)(1 + 3i)…(1 + ni)= (x+ iy), show that :

$$2.5.10\ldots\ (1+n^2) = x^2 + y^2$$

18 If $z_{1,}z_2$ are complex number such that $\frac{2z_1}{3z_2}$ is purely imaginary number, then find $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$

19 Find the square roots of the following

(i) 7 – 24i (ii) 5 + 12i

20 Find the value of $x^3 + 7x^2 - x + 16$, when x = 1 + 2i.

21 If $x = -5 + 2\sqrt{-4}$, find the value of $x^4 + 9x^3 + 35^2 - x + 4$

22 If z is a complex number such that $|z| = 1$, prove that $\left(\frac{z-1}{z+1}\right)$ is purely imaginary. What will be your conclusion if $z = 1$?

23 If $z = x + iy$ and $w = \frac{1-iz}{z-i}$, show that $|w| = 1 \Rightarrow z$ is purely real.

24 Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

25 Show that a real value of x will satisfy the equation $\frac{1-ix}{1+ix} = a - ib$ if $a^2 + b^2 = 1$, where a, b are real.

26 If α and β are different complex numbers with $|\beta| = 1$, find $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|$.

27 If $z_1 = 2 - i, z_2 = 1 + i$, find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$

28 If $z_1 = 2 - i, z_2 = -2 + i$, find

(i) $\mathrm{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right)$ (ii) $\mathrm{Im}\left(\frac{1}{z_1\overline{z}_1}\right)$

29 Find the real values of θ for which the complex number r $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real.

30 Express the following complex numbers in the polar from:

(i) $\frac{1+i}{1-i}$ (ii) $\frac{2+6\sqrt{3}i}{5+\sqrt{6}i}$

31 Put the following in the from $r(\cos\theta + i\sin\theta)$, where r is a positive real numbers and $-\pi < \theta \le \pi : \frac{(1+7i)}{(2-i)^2}$

32 Find the modulus and argument of the following complex numbers and convert them in polar from:

(i) $\dfrac{1+2i}{1-3i}$ (ii) $\dfrac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ (iii) $\dfrac{1+3i}{1-2i}$

33 Express the following complex numbers in the form $r\ (\cos\theta+i\sin\theta)$:

(i) $1+i\tan\alpha$ (ii) $\tan\alpha-i$ (iii) $1-\sin\alpha+i\cos\alpha$