CHAPTER – 5
Complex Numbers

"Complex number is the combination of real and imaginary numbers"

5.1 Basic concepts of complex number

(1) **Definition:** A number of the form $x+iy$, where $x, y \in R$ and $i=\sqrt{-1}$ is called a complex number and 'i' is called iota.

A complex number is usually denoted by z and the set of complex number is denoted by C.

i.e., $C=\{x+iy : x \in R, y \in R, i=\sqrt{-1}\}$

For example, $5+3i, -1+i, 0+4i, 4+0i$ etc. are complex numbers.

(i) Euler was the first mathematician to introduce the symbol i (iota) for the square root of – 1 with property $i^2=-1$. He also called this symbol as the imaginary unit.

(ii) For any positive real number a, we have

$\sqrt{-a}=\sqrt{-1\times a}=\sqrt{-1}\sqrt{a}=i\sqrt{a}$

(iii) The property $\sqrt{a}\sqrt{b}=\sqrt{ab}$ is valid only if at least one of a and b is non-negative. If a and b are both negative then $\sqrt{a}\sqrt{b}=-\sqrt{|a|.|b|}$.

(2) **Integral powers of iota (i):** Since $i=\sqrt{-1}$ hence we have $i^2=-1$, $i^3=-i$ and $i^4=1$. To find the value of $i^n (n>4)$, first divide n by 4. Let q be the quotient and r be the remainder.

i.e., $n=4q+r$ where $0\le r\le 3$

$i^n=i^{4q+r}=(i^4)^q.(i)^r=(1)^q.(i)^r=i^r$

In general we have the following results $i^{4n}=1, i^{4n+1}=i, i^{4n+2}=-1$, $i^{4n+3}=-i$, where n is any integer.

Real and Imaginary parts of a complex number

If x and y are two real numbers, then a number of the form $z=x+iy$ is called a complex number. Here 'x' is called the real part of z and 'y' is known as the imaginary part of z. The real part of z is denoted by $Re(z)$ and the imaginary part by Im(z).

If $z=3-4i$, then Re(z) = 3 and Im(z) = – 4.

A complex number z is purely real if its imaginary part is zero *i.e.*, Im(z) = 0 and purely imaginary if its real part is zero *i.e.*, Re(z) = 0.