5.2 Algebraic operations with complex numbers

Let two complex numbers *bc* $z_1 = a + ib$ and $z_2 = c + id$

Addition ($z_1 + z_2$) : $(a+ib)+(c+id)=(a+c)+i(b+d)$

Subtraction $(z_1 - z_2)$: $(a+ib)-(c+id)=(a-c)+i(b-d)$

Multiplication $(z_1 . z_2)$: $(a+ib)(c+id)=(ac-bd)+i(ad+bc)$

Division (z_1 / z_2) : $\frac{a+ib}{c+id}$

(where at least one of *c* and *d* is non-zero)

$\frac{a+ib}{c+id}=\frac{(a+ib)}{(c+id)}.\frac{(c-id)}{(c-id)}$ (Rationalization)

$\frac{a+ib}{c+id}=\frac{(ac+bd)}{c^2+d^2}+\frac{i(bc-ad)}{c^2+d^2}.$

Properties of algebraic operations on complex numbers: Let z_1, z_2 and z_3 are any three complex numbers then their algebraic operations satisfy following properties :

(i) Addition of complex numbers satisfies the commutative and associative properties

i.e., $z_1 + z_2 = z_2 + z_1$ and $(z_1+z_2)+z_3 = z_1 + (z_2 + z_3)$.

(ii) Multiplication of complex numbers satisfies the commutative and associative properties.

i.e., $z_1 z_2 = z_2 z_1$ and $(z_1 z_2) z_3 = z_1 (z_2 z_3)$.

(iii) Multiplication of complex numbers is distributive over addition

i.e., $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$ and $(z_2 + z_3) z_1 = z_2 z_1 + z_3 z_1$.

Equality of two complex numbers

Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal if and only if their real and imaginary parts are separately equal.

i.e., $z_1 = z_2 \Leftrightarrow x_1 + iy_1 = x_2 + iy_2 \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$.

Complex numbers do not possess the property of order *i.e.*, $(a+ib) < (\text{or}) > (c+id)$ is not defined. For example, the statement $(9+6i) > (3+2i)$ makes no sense.