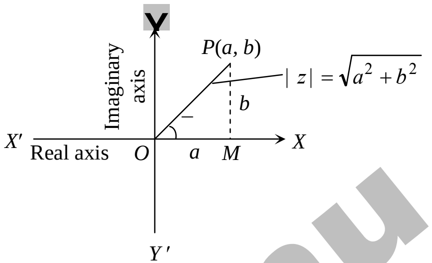
5.7 Various representations of a complex number

A complex number can be represented in the following form:

(1) Geometrical representation (Cartesian representation): The complex number $z = a + ib = (a, b)$ is represented by a point P whose coordinates are referred to rectangular axes XOX' and YOY' which are called real and imaginary axis respectively. This plane is called argand plane or argand diagram or complex plane or Gaussian plane.



Distance of any complex number from the origin is called the modulus of complex number and is denoted by |z|, *i.e.*, $|z| = \sqrt{a^2 + b^2}$.

Angle of any complex number with positive direction of *x*–axis is called amplitude or argument of *z*.

i.e., $amp\,(z) = arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$

(2) Trigonometrical (Polar) representation: In Δ *OPM*, let $OP = r$, then $a = r\cos\theta$ and $b = r\sin\theta$. Hence *z* can be expressed as $z = r(\cos\theta + i\sin\theta)$

where $r = |z|$ and θ = principal value of argument of *z*.

For general values of the argument $z = r[\cos(2n\pi + \theta) + i\sin(2n\pi + \theta)]$

Sometimes $(\cos\theta + i\sin\theta)$ is written in short as $cis\,\theta$.

(3) Vector representation: If *P* is the point (*a*, *b*) on the argand plane corresponding to the complex number $z = a + ib$.

Then $\overrightarrow{OP} = a\hat{i} + b\hat{j}$, $\therefore |\overrightarrow{OP}| = \sqrt{a^2 + b^2} = |z|$ and

arg (*z*) = direction of the vector $\overrightarrow{OP} = \tan^{-1}\left(\frac{b}{a}\right)$

(4) Eulerian representation (Exponential form): Since we have $e^{i\theta} = \cos\theta + i\sin\theta$ and thus *z* can be expressed as $z = re^{i\theta}$, where $|z| = r$ and $\theta =$ *arg* (*z*).

Logarithm of a complex number

$$\log(x + iy) = \log_e(re^{i\theta}) = \log_e r + \log_e e^{i\theta} = \log_e r + i\theta$$

$$= \log_e \sqrt{(x^2 + y^2)} + i\tan^{-1}\left(\frac{y}{x}\right)$$

$$\boxed{\log_e(z) = \log_e |z| + i\,amp\,z}$$