CHAPTER – 6
Linear Inequations

EXAMPLE - 1

Solve 24x < 100, when

(i) x is a natural number. (ii) x is an integer.

Solution: We have, 24x < 100

⇒ $\frac{24x}{24} < \frac{100}{24}$ [On dividing both sides by 24]

⇒ x < $\frac{25}{6}$

(i) Given that x is a natural number, i.e., x ∈ N

Hence, the solution set of given inequality = $\left\{x \in N : x < \frac{25}{6}\right\}$ = {1, 2, 3, 4}.

(ii) Given that x is an integer, i.e., x ∈ Z.

Hence, the solution set of given inequality = $\left\{x \in Z : x < \frac{25}{6}\right\}$

= {..., -2, -1, 0, 1, 2, 3, 4}.

EXAMPLE - 2

Solve -12x > 30, when

(i) x is a natural number. (ii) x is an integer.

Solution: We have, -12x > 30

⇒ $\frac{-12x}{-12} < \frac{30}{-12}$ [On dividing both sides by -12]

⇒ x < $-\frac{5}{2}$

(i) Given that x is a natural number, i.e., x ∈ N.

Hence, the solution set of given inequality = $\left\{x \in N : x < -\frac{5}{2}\right\}$ = { }

(ii) Given that x is an integer, i.e., x ∈ Z.

Hence, the solution set of given inequality = $\left\{x \in Z : x < -\frac{5}{2}\right\}$

= {..., -5, -,4 -3}

EXAMPLE - 3

Solve 3x + 8 > 2, when

(i) x is an integer. (ii) x is a real number.

Solution: We have, 3x + 8 > 2

⇒ 3x + 8 - 8 > 2 - 8 [On subtracting 8 from both Sides]

⇒ 3x > -6

⇒ $\frac{3x}{x} > \frac{-6}{3}$ [On dividing both sides by 3]

⇒ x > -2

(i) Given that x is an integer, i.e., x ∈ Z.

Hence, the solution set of given inequality = {x ∈ Z ; x > -2} = {-1, 0, 1, ...}.

(ii) Given that x is a real number, i.e., $x \in R$.
Hence, the solution set of given inequality = $\{x \in R ; x > -2\} = (-2, \infty)$

EXAMPLE - 4

Solve 5x - 3 < 3x + 1, when
(i) x is an integer. (ii) x is a real number.

Solution: We have, $5x - 3 < 3x + 1$

$\Rightarrow$ $5x - 3 - 3x < 3x + 1 - 3x$ [On subtracting 3x from both sides]

$\Rightarrow$ $2x - 3 < 1$

$\Rightarrow$ $2x - 3 + 3 < 1 + 3$ [On adding 3 on both sides]

$\Rightarrow$ $2x < 4$

$\Rightarrow$ $\frac{2x}{2} < \frac{4}{2}$ [On dividing both sides by 2]

$\Rightarrow$ $x < 2$

(i) Given that x is an integer, i.e., $x \in Z$.
Hence, the solution set of given inequality = $\{x \in Z : x < 2\}$
$= \{..., -2, -1, 0, 1\}$.

(ii) Given that x is a real number, i.e., $x \in R$.
Hence, the solution set of given inequality = $\{x \in R : x < 2\} = (-\infty, 2)$.

EXAMPLE - 5

Solve the following inequalities for real x:

(i) $3(2 - x) \geq 2(1 - x)$. (ii) $\frac{5-2x}{3} \leq \frac{x}{6} - 5$

(iii) $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6)$ (iv) $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$

Solution:

(i) The solution set of given inequality = $\{x \in R : x \leq 4\} = (-\infty, 4]$.

(ii) The solution set of given inequality = $\{x \in R : x \geq 8\} = [8, \infty)$.

(iii) The solution set of given inequality = $\{x \in R : x \leq 120\} = (-\infty, 120]$

(iv) The solution set of given inequality = $\{x \in R : x \leq 2\} = (-\infty, 2]$

Example - 6

Solve the following inequalities and show the graph of the solution (in each case) on number line:

(i) $7x + 3 < 5x + 9$ (ii) $3x - 2 \leq 2x + 1$

(iii) $3(1 - x) < 2(x + 4)$ (iv) $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

Solution:

(i) The solution set of given inequality = $\{x \in R : x < 3\} = (-\infty, 3)$.

(ii) The solution set of given inequality = $\{x \in R : x \leq 3\} = (-\infty, 3]$.

(iii) The solution set of given inequality = $\{x \in R : x > -1\} = (-1, \infty)$

(iv) The solution set of given inequality = $\{x \in R : x \geq 1\} = [1, \infty)$

6.1.1 Compound Inequalities

Example - 1

Solve the following inequalities for real x:

(i) $-8 \le 5x - 3 < 7$ (ii) $6 \le -3(2x - 4) < 12$.

(iii) $-3 \le 4 - \frac{7x}{2} \le 18$ (iv) $-15 < \frac{3(x-2)}{5} \le 0$

Solution:

(i) The solution set of given inequality = $\{x \in R : x < 2\} = [-1, 2)$.

(ii) The solution set of given inequality = $\{x \in R : 1 \ge x > 0\} = (0, 1]$.

(iii) The solution set of given inequality = $\{x \in R : 2 \ge x \ge -4\} = [-4, 2]$

(iv) The solution set of given inequality = $\{x \in R : -23 < x \le 2\} = (-23, 2]$

Example - 2

Solve the following inequalities and show the graph of the solution (in each case) on number line:

(i) $3x - 7 > 2(x - 6)$, $6 - x > 11 - 2x$

(ii) $5(2x - 7) - 3(2x + 3) \le 0$, $2x + 19 \le 6x + 47$

Solution:

(i) The solution set of given inequality = $\{x \in R : x > -5, x > 5\} = (5, \infty)$.

(ii) The solution set of given inequality = $\{x \in R : x \le 11, x \ge -7\} = [-7, 11]$.

6.1.2 Absolute-Value Inequalities

Example - 1

Solve the real x: $|2x| < 4$

Solution:

Case I When $x \ge 0$

In this case, the solution set = $\{x \in R : x \ge 0 \text{ and } x < 2\} = [0, 2)$

Case II When $x < 0$

In this case, the solution set = $\{x \in R : x < 0 \text{ and } x > -2\} = (-2, 0)$

Hence, the solution set of given inequality = $[0, 2) \cup (-2, 0) = (-2, 2)$

Example - 2

Solve the real x: $|x - 2| \le 3$

Solution:

Case I When $x \ge 2$

In this case, the solution set = $\{x \in R : x \ge 2 \text{ and } x \le 5\} = [2, 5]$

Case II When x < 2

In this case, the solution set = {x ∈ R : x < 2 and x ≥ -1} = [-1, 2)

Hence, the solution set of given inequality = [2, 5] ∪ [-1, 2) = [-1, 5]

Example - 3

Solve the real x: |2x - 5| > 1

Solution:

Case I When x ≥ $\frac{5}{2}$

In this case, the solution set = {x ∈ R : x ≥ $\frac{5}{2}$ and x > 3} = (3, ∞)

Case II When x < $\frac{5}{2}$

In this case, the solution set = {x ∈ R : x < $\frac{5}{2}$ and x < 2} = (-∞, 2)

Hence, the solution set of given inequality = (3, ∞) ∪ (-∞, 2)

Example - 4

Solve the real x: |3 - 4x| ≥ 9

Solution:

Case I When x ≤ $\frac{3}{4}$

In this case, the solution set = {x ∈ R : x ≤ $\frac{3}{4}$ and x ≤ $-\frac{3}{2}$} = (-∞, $-\frac{3}{2}$)

Case II When x > $\frac{3}{4}$

In this case, the solution set = {x ∈ R : x > $\frac{3}{4}$ and x ≥ 3} = [3, ∞)

Hence, the solution set of given inequality = $\left\{x \in R : x \geq 3 \text{ or } x \leq -\frac{3}{2}\right\}$

$= \left(-\infty, -\frac{3}{2}\right] \cup [3, \infty)$

HOTS (Higher Order Thinking Skills)

Example H - 1

Solve the real x: $1 \le |x - 2| \le 3$

Solution:

Case I When $x \ge 2$

In this case, the solution set = $\{x \in R : x \ge 2 \text{ and } 3 \le x \le 5\} = [3, 5]$

Case II When $x < 2$

In this case, the solution set = $\{x \in R : x < 2 \text{ and } -1 \le x \le 1\} = [-1, 1]$

Hence, the solution set of given inequality = $[3, 5] \cup [-1, 1]$

Example H - 2

Solve the real x: $|x + 1| + |x| > 3$

Solution:

Case I When $x < -1$

In this case, the solution set = $\{x \in R : x < -1 \text{ and } x < -2\}$
$= (-\infty, -2)$

Case II When $-1 \le x < 0$

In this case, the solution set = ϕ

Case III When $x \ge 0$

In this case, the solution set = $\{x \in R : x \ge 0 \text{ and } x > 1\} = (1, \infty)$

Hence, the solution set of given inequality = $(-\infty, -2) \cup (1, \infty)$

Example H - 3

Solve the real x: $\frac{|x+3|+x}{x+2} > 1, x \ne -2$

Solution:

Case I When $x \ge -3$

In this case, the solution set = $\{x \in R : x \ge -3 \text{ and } x \in (-\infty, -2) \cup (-1, \infty)\}$
$= [-3, -2) \cup (-1, \infty)$.

Case II When $x < -3$

In this case, the solution set = $\{x \in R : x < -3 \text{ and } (-5, -2)\} = (-5, -3)$

Hence, the solution set of given inequality $= [-3, -2) \cup (-1, \infty) \cup (-5, -3)$
$= (-5, -2) \cup (-1, \infty)$

Exercise 6.1

1. Solve 30x < 200, when
   (i) x is a natural number. (ii) x is an integer.
2. Solve 5x - 3 < 7, when
   (i) x is an integer. (ii) x is a real number.
3. Solve the following inequalities for real x:
   (i) 4x + 3 < 6x + 7. (ii) 3(x - 1) ≤ 2(x - 3).
   (iii) $\frac{x}{3} > \frac{x}{2} + 1$. (iv) $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$
   (v) 2(2x + 3) - 10 < 6(x - 2).
4. Solve the following inequalities and show the graph of the solution (in each case) on number line:
   (i) 5x - 3 ≥ 3x - 5. (ii) $\frac{x}{2} < \frac{5x-2}{3} - \frac{7x-3}{5}$
5. Solve the following inequalities for real x:
   (i) 3x - 7 > 5x - 1. (ii) 37 - (3x + 5) > 9x - 8(x - 3).
   (iii) $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$ (iv) x + $\frac{x}{2} + \frac{x}{3} < 11$.
   (v) 2 ≤ 3x - 4 ≤ 5. (vi) $-5 \leq \frac{5-3x}{2} \leq 8$.
   (vii) $-12 < 4 + \frac{3x}{5} \leq 2$. (viii) $7 \leq \frac{3x+11}{2} \leq 11$.
6. Solve the following inequalities and show the graph of the solution (in each case) on number line:
   (i) 3x - 7 < 5 + x, 11 - 5x ≤ 1
   (ii) 5x + 1 > -24, 5x - 1 < 24.
   (iii) 2(x - 1) < x + 5, 3(x + 2) > 2 - x.
7. Solve for real x: |x - 1| > 5.
8. Solve for real x: |x + 2| ≤ 9.
9. Solve for real x: |x + 3| ≥ 10.
10. Solve for real x: |x - 1| ≤ 5, |x| ≥ 2.

Answers 6.1

1. (i) {1, 2, 3, 4, 5, 6} (ii) {... , -3, -2, -1, 0, 1, 2, 3, 4, 5, 6}
2. (i) {... , -2, -1, 0, 1} (ii) (-∞, 2)
3. (i) (-2, ∞) (ii) (-∞, -3] (iii) (-∞, -6)
   (iv) (4, ∞) (v) (4, ∞)
4. (i) [-1, ∞) (ii) $(-\infty, -\frac{2}{7})$
5. (i) (-∞, -3) (ii) (-∞, 2) (iii) (-∞, 2] (iv) (-∞, 6)
   (v) [2, 3] (vi) $\left[-\frac{11}{3}, 5\right]$ (vii) $\left(-\frac{80}{3}, -\frac{10}{3}\right]$ (viii) $\left[1, \frac{11}{3}\right]$
6. (i) [2, 6) (ii) (-5, 5) (iii) (-1, 7)
7. (-∞, -4) ∪ (6, ∞)

8. [-11, 7]
9. $(-\infty, -13] \cup [7, \infty)$
10. $[-4, -2] \cup [2, 6]$