CHAPTER – 7
Permutations and Combinations

Permutations

7.1 Introduction

(1) **The Factorial:** Factorial notation: Let n be a positive integer. Then, the continued product of first n natural numbers is called factorial n, to be denoted by $n\,!$ or $\underline{|n}$.

Also, we define $0\,! = 1$.

when n is negative or a fraction, $n\,!$ is not defined.

Thus, $n\,! = n\,(n-1)\,(n-2)\,......3.2.1$.

Definition of permutation

The ways of arranging or selecting a smaller or an equal number of persons or objects at a time from a given group of persons or objects with due regard being paid to the order of arrangement or selection are called the (different) *permutations*.

For example : Three different things a, b and c are given, then different arrangements which can be made by taking two things from three given things are ab, ac, bc, ba, ca, cb.

Therefore the number of permutations will be 6.

Number of permutations without repetition

(1) Arranging n objects, taken r at a time equivalent to filling r places from n things.

r-places :	1	2	3	4		r
	n	$(n-1)$	$(n-2)$	$(n-3)$		$n-(r-1)$

Number of choices :

The number of ways of arranging = The number of ways of filling r places.

$$= n(n-1)(n-2).......(n-r+1)$$

$$= \frac{n(n-1)(n-2).....(n-r+1)((n-r)!)}{(n-r)!} = \frac{n\,!}{(n-r)!} = {}^nP_r$$

(2) The number of arrangements of n different objects taken all at a time = ${}^nP_n = n!$

(i) ${}^nP_0 = \frac{n!}{n!} = 1; {}^nP_r = n.{}^{n-1}P_{r-1}$

(ii) $0! = 1; \frac{1}{(-r)!} = 0$ or $(-r)! = \infty \ (r \in N)$

Number of permutations with repetition

(1) The number of permutations (arrangements) of n different objects, taken r at a time, when each object may occur once, twice, thrice,........upto r times in any arrangement = The number of ways of filling r places where each place can be filled by any one of n objects.

r – places :

1	2	3	4	r
n	n	n	n	n

Number of choices :

The number of permutations = The number of ways of filling r places = $(n)^r$.

(2) The number of arrangements that can be formed using n objects out of which p are identical (and of one kind) q are identical (and of another kind), r are identical (and of another kind) and the rest are distinct is $\frac{n!}{p!q!r!}$.