7.2 Conditional permutations

(1) Number of permutations of n dissimilar things taken r at a time when p particular things always occur = ${}^{n-p}C_{r-p}\, r!$.

(2) Number of permutations of n dissimilar things taken r at a time when p particular things never occur = ${}^{n-p}C_{r}\, r!$.

(3) The total number of permutations of n different things taken not more than r at a time, when each thing may be repeated any number of times, is $\frac{n(n^r-1)}{n-1}$.

(4) Number of permutations of n different things, taken all at a time, when m specified things always come together is $m!\times(n-m+1)!$.

(5) Number of permutations of n different things, taken all at a time, when m specified things never come together is $n!-m!\times(n-m+1)!$.

(6) Let there be n objects, of which m objects are alike of one kind, and the remaining $(n-m)$ objects are alike of another kind. Then, the total number of mutually distinguishable permutations that can be formed from these objects is $\frac{n!}{(m!)\times(n-m)!}$.

The above theorem can be extended further *i.e.*, if there are n objects, of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of 3rd kind;.....; p_r are alike of r^{th} kind such that $p_1+p_2+.....+p_r=n$; then the number of permutations of these n objects is $\frac{n!}{(p_1!)\times(p_2!)\times.....\times(p_r!)}$.