7.3 Circular permutations

In circular permutations, what really matters is the position of an object relative to the others.

Thus, in circular permutations, we fix the position of the one of the objects and then arrange the other objects in all possible ways.

There are two types of circular permutations :

(i) The circular permutations in which clockwise and the anticlockwise arrangements give rise to different permutations, *e.g.* Seating arrangements of persons round a table.

(ii) The circular permutations in which clockwise and the anticlockwise arrangements give rise to same permutations, *e.g.* arranging some beads to form a necklace.

Difference between clockwise and anti-clockwise arrangement : If anti-clockwise and clockwise order of arrangement are not distinct *e.g.*, arrangement of beads in a necklace, arrangement of flowers in garland etc. then the number of circular permutations of n distinct items is $\frac{(n-1)!}{2}$.

(i) Number of circular permutations of n different things, taken r at a time, when clockwise and anticlockwise orders are taken as different is $\frac{^nP_r}{r}$.

(ii) Number of circular permutations of n different things, taken r at a time, when clockwise and anticlockwise orders are not different is $\frac{^nP_r}{2r}$.

Theorems on circular permutations

Theorem (i): The number of circular permutations of n different objects is $(n-1)!$.

Theorem (ii): The number of ways in which n persons can be seated round a table is $(n-1)!$.

Theorem (iii): The number of ways in which n different beads can be arranged to form a necklace, is $\frac{1}{2}(n-1)!$.