Chapter - 8

ASSIGNMENT

1 By using binomial theorem, expand:

(i) $(1 + x + x^2)^3$ (ii) $(1 - x + x^2)^4$

2 Using binomial theorem, expand $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$, $x \neq 0$.

3 Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem.

4 Using binomial theorem, expand $\{(x + y)^5 + (x - y)^5\}$ and hence find the value of $\left\{(\sqrt{2}+1)^5+(\sqrt{2}-1)^5\right\}$.

5 If O be the sum of odd terms and E that of even terms in the expansion of $(x + a)^n$, prove that:

(i) $O^2 - E^2 = (x^2 - a^2)^n$ (ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$

(iii) $2(O^2 + E^2) = (x + a)^{2n} + (x - a)^{2n}$

6 Which is larger $(1.01)^{1000000}$ or 10,000?

7 Write down the binomial expansion of $(1 + x)^{n+1}$, when x = 8. Deduce that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

8 Using binomial theorem, prove that $(101)^{50} > 100^{50} + 99^{50}$

9 Evaluate the following:

(i) $\left(\sqrt{x+1}+\sqrt{x-1}\right)^6+\left(\sqrt{x+1}-\sqrt{x-1}\right)^6$ (ii) $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$

10 $\left\{a^2+\sqrt{a^2-1}\right\}^4+\left\{a^2-\sqrt{a^2-1}\right\}^4$

11 Using binomial theorem, prove that $3^{2n+2} - 8n - 9$ is divisible by 64, $n \in N$

12 Find the value of $(1.01)^{10} + (1 - 0.01)^{10}$ correct to 7 places of decimal.

13 Find the 10th term in the binomial expansion of $\left(2x^2+\frac{1}{x}\right)^{12}$

14 Find the 9th term in the expansion of $\left(\frac{x}{a}-\frac{3a}{x^2}\right)^{12}$

15 Find16 13th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$.

16 If the coefficients of (2r + 1)th term and (r + 2)th term in the expansion of $(1 + x)^{43}$ are equal, find r.

17 If the coefficients of (2r + 4)th and (r – 2) term in the expansion of $(1 + x)^{18}$ are equal, find r.

18 Find the 4^{th} term from the end in the expansion of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^{9}$.

19 Which term in the expansion of $\left\{\left(\frac{x}{\sqrt{y}}\right)^{1/3}+\left(\frac{y}{x^{1/3}}\right)^{1/2}\right\}^{21}$ contains x and y to one and the same power?

20 Find the 8^{th} term in the expansion of $(x^{3/2} y^{1/2} – x^{1/2} y^{3/2})^{10}$.

21 Find the 11th term from the end in the expansion of $\left(2x-\frac{1}{x^2}\right)^{25}$

22 Find the coefficient of x^{10} in the binomial expansion of $\left(2x^2-\frac{3}{x}\right)^{11}$, when $x \neq 0$.

23 Find the coefficient of x^{40} in the expansion of $(1 + 2x + x^2)^{27}$.

24 Find the coefficients of x^{32} and x^{-17} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$

25 Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}-\frac{1}{\sqrt[4]{3}}\right)^{n}$ is $\sqrt{6}:1$.

26 Find a, if 17th and 18th terms in the expansion of $(2 + a)^{50}$ are equal.

27 If the third term in the expansion of $\left(\frac{1}{x}+x^{\log_{10} x}\right)^{5}$ is 1000, then find x.

28 If the fourth term in the expansion of $\left\{\sqrt{\frac{1}{x^{\log x+1}}}+x^{\frac{1}{12}}\right\}^6$ is equal to 200 and x > 1, then find x.

29 For what value of x is the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{-1/8\log_3(5^{x-1}+1)}\right\}^{10}$ is equal to 180 ?

30 Find the middle terms in the expansion of $\left(3x-\frac{x^3}{6}\right)^7$.

31 Find the value of α for which the coefficients of the middle terms in the expansions of $(1+\alpha x)^4$ and $(1-\alpha x)^6$ are equal, find α.

32 Find the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ and x^7 in $\left(ax-\frac{1}{bx^2}\right)^{11}$ and find the relation between a and b so that these coefficients are equal.

33 If x^p occurs in the expansion of $\left(x^2+\frac{1}{x}\right)^{2n}$, prove that its coefficient is $\left[\frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+P}{3}\right)!}\right]$

34 Find the coefficient of x^5 in the expansion of the product $(1+2x)^6(1-x)^7$.

35 Find the coefficient of x^n in the expansion of $(1+x)(1-x)^n$.

36 If the coefficients of x and x^2 in the expansion of $(1+x)^m(1-x)^n$ are 3 and - 6 respectively. Find the values of m and n.

37 Prove that the term independent of x in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is $\frac{1.3.5...(2n-1)}{n!}.2^n$.

38 Find the term independent of x in the expansion of $\left(3x^2-\frac{1}{2x^3}\right)^{10}$

39 Find the term independent of x in the expansion of

(i) $\left(x-\frac{1}{x}\right)^{12}$ (ii) $\left(2x-\frac{1}{x}\right)^{10}$

40 Find the coefficient of the term independent of x in the expansion of

$$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$$

41 Find the greatest value of the term independent of x in the expansion of $\left(x\sin\alpha+\frac{\cos\alpha}{x}\right)^{10}$,where $\alpha \in$ R.

42 If the fourth term in the expansion of $\left(ax+\frac{1}{x}\right)^{n}$ is $\frac{5}{2}$, then find the values of a and n.

43 Find the value of a so that the term independent of x in $\left(\sqrt{x}+\frac{a}{x^2}\right)^{10}$ 405.

44 If the coefficients of $(r - 5)^{th}$ and $(2r - l)^{th}$ terms in the expansion of $(1+ x)^{34}$ are equal, find r.

45 Find the coefficient of x^5 in the expansion of $(1 + x)^{21} + (1 + x)^{22} + ... + (1 + x)^{30}$

46 The coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 7 : 42. Find n.

47 If the coefficients of a^{r-1}, a^r, a^{r+1} in the binomial expansion of $(1 + a)^n$ are in A.P., prove that $n^2 - n(4r + 1) + 4r^2 - 2 = 0$.

48 If a_1 a_2, a_3, a_4 be the coefficients of four consecutive terms in the expansion of $(1 + x)^n$, then prove that: $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$.

49 The 3^{rd}, 4^{th} and 5^{th} terms in the expansion of $(x + a)^n$ are respectively 84, 280 and 560, find the values of x, a and n.

50 If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ be 76, 95 and 76, find n.