CHAPTER – 8
Binomial Theorem

8.1 INTRODUCTION

Binomial expression

An algebraic expression consisting of two terms
with +*ve* or – *ve* sign between them is called a binomial expression.

For example **:** $(a+b), (2x-3y), \left(\frac{p}{x^2}-\frac{q}{x^4}\right), \left(\frac{1}{x}+\frac{4}{y^3}\right)$ etc.

Binomial theorem for positive integral index

The rule by which any power of binomial can be expanded is called the binomial theorem.

If *n* is a positive integer and *x*, $y \in C$ then

$$(x+y)^n = {}^nC_0 x^{n-0} y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots\ldots + {}^nC_r x^{n-r} y^r + \ldots\ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

i.e., $(x+y)^n = \sum_{r=0}^{n} {}^nC_r . x^{n-r} . y^r$ .....(i)

Here ${}^nC_0, {}^nC_1, {}^nC_2, \ldots\ldots {}^nC_n$ are called binomial coefficients and ${}^nC_r = \frac{n!}{r!(n-r)!}$ for $0 \le r \le n$.

Some important expansions

(1) Replacing y by $-y$ in (i), we get,

$$(x-y)^n = {}^nC_0 x^{n-0} y^0 - {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 - \ldots\ldots\ldots + (-1)^r\, {}^nC_r x^{n-r} y^r + \ldots\ldots + (-1)^n\, {}^nC_n x^0 y^n$$

i.e., $(x-y)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r x^{n-r} y^r$

The terms in the expansion of $(x-y)^n$ are alternatively positive and negative, the last term is positive or negative according as *n* is even or odd.

(2) Replacing *x* by 1 and *y* by *x* in equation (i) we get,
$$(1+x)^n = {}^nC_0 x^0 + {}^nC_1 x^1 + {}^nC_2 x^2 + \ldots\ldots + {}^nC_r x^r + \ldots\ldots + {}^nC_n x^n$$

i.e., $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$

This is expansion of $(1+x)^n$ in ascending power of *x*.

(3) Replacing *x* by 1 and *y* by – *x* in (i) we get,

$$(1-x)^n = {}^nC_0 x^0 - {}^nC_1 x^1 + {}^nC_2 x^2 - \ldots\ldots + (-1)^r\, {}^nC_r x^r + \ldots\ldots + (-1)^n\, {}^nC_n x^n$$

i.e., $(1-x)^n = \sum_{r=0}^{n} (-1)^r \, {}^nC_r x^r$

(4) $(x+y)^n + (x-y)^n = 2[{}^nC_0 x^n y^0 + {}^nC_2 x^{n-2} y^2 + {}^nC_4 x^{n-4} y^4 + ...]$ and

$(x+y)^n - (x-y)^n = 2[{}^nC_1 x^{n-1} y^1 + {}^nC_3 x^{n-3} y^3 + {}^nC_5 x^{n-5} y^5 + ...]$

(5) The coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^n$ is ${}^nC_r$.

(6) The coefficient of x^r in the expansion of $(1+x)^n$ is ${}^nC_r$.