8.2 General term

The general term of the expansion is $(r+1)^{th}$ term usually denoted by T_{r+1} and $T_{r+1} = {}^nC_r x^{n-r} y^r$

- In the binomial expansion of $(x-y)^n, T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$
- In the binomial expansion of $(1+x)^n, T_{r+1} = {}^nC_r x^r$
- In the binomial expansion of $(1-x)^n, T_{r+1} = (-1)^r \, {}^nC_r x^r$
- In the binomial expansion of $(x+y)^n$, the p^{th} term from the end is $(n-p+2)^{th}$ term from beginning.

Independent term or Constant term

Independent term or constant term of a binomial expansion is the term in which exponent of the variable is zero.

Condition : $(n-r)$ [Power of x] + r [Power of y] = 0, in the expansion of $[x+y]^n$.

Number of terms in the expansion of $(a+b+c)^n$ and $(a+b+c+d)^n$

$(a+b+c)^n$ can be expanded as : $(a+b+c)^n = \{(a+b)+c\}^n$

$$= (a+b)^n + {}^nC_1 (a+b)^{n-1}(c)^1 + {}^nC_2 (a+b)^{n-2}(c)^2 + \ldots\ldots + {}^nC_n c^n$$

$$= (n+1)\text{term} + n\,\text{term} + (n-1)\text{term} + \ldots + 1\,\text{term}$$

$\therefore$ Total number of terms = $(n+1)+(n)+(n-1)+\ldots\ldots+1 = \frac{(n+1)(n+2)}{2}$.

Similarly, number of terms in the expansion of

$$(a+b+c+d)^n = \frac{(n+1)(n+2)(n+3)}{6}.$$