Chapter - 9

ASSIGNMENT

1 Show that the sequence log a, $\log\left(\frac{a^2}{b}\right), \log\left(\frac{a^3}{b^2}\right), \log\left(\frac{a^4}{b^3}\right), \ldots\ldots$ forms on A.P.

2 If $\log_2$, $\log(2^x - 1)$ and $\log(2^x + 3)$ are in A.R, write the value of x.

3 Write the common difference of an A.P. the sum of whose first n terms is $\frac{P}{2}n^2 + Qn$.

4 Find the number of terms common to the two A.P.'s: 3, 7, 11, ... 407 and 2, 9, 16, .. ., 709.

5 In the arithmetic progressions 2, 5, 8,... upto 50 terms, and 3, 5, 7, 9, ... upto 60 terms, find how many terms are identical

6 If $a_1, a_2, a_3, \ldots, a_n$ are in A.P., where $a_i > 0$ for all i, show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

7 If $a_1, a_2, a_3, \ldots, a_n$ be an A.P. of non-zero terms, prove that

$$\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \ldots + \frac{1}{a_{n-1}a_n} = \frac{n-1}{a_1a_n}.$$

8 The angles of a quadrilateral are in A.P. whose common difference is 10°. Find the angles.

9 If the sum of three numbers in A.P. is 24 and their product is 440, find the numbers.

10 Find the sum of first 24 terms of the A.P. $a_1, a_2, a_3, \ldots$, if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} + = 225$.

11 Let S_n denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$, then prove that $\frac{S_{3n}}{S_n} = 6$.

12 The number of terms of an A.P. is even; the sum of odd terms is 24, of the even terms is 30, and the last term exceeds the first by $10\frac{1}{2}$, find the number of terms and the series.

13 If the sum of n terms of an A.P. is $nP + \frac{1}{2}n(n - 1)Q$, where P and Q are constants, find the common difference.

14 If n arithmetic means are inserted between 20 and 80 such that the ratio of first mean to the last mean is 1:3, then find the value of n.

15 The sum of two numbers is $\frac{13}{6}$. An even number of arithmetic means are being inserted between them and their sum exceeds their number by 1. Find the number of means inserted.

16 There are n A.M.s between 3 and 17. The ratio of the last mean to the first mean is 3:1. Find the value of n.

17 If x, y, z are in A.P. and A_1 is the A.M. of x and y and A_2 is the A.M. of y and z, then prove that the A.M. of A_1 and A_2 is y.

18 Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by 1/2 km each succeeding hour. After how many hours will the second car overtake the first car if both cars go non-stop ?

19 Along a road He an odd number of stones placed at intervals of 10 metres. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried the job with one of the end stones by carrying them in succession. In carrying all the stones he covered a distance of 3 km. Find the number of stones.

20 Shamshad Ali buys a scooter for Rs 22000. He pays Rs 4000 cash and a grees to pay the balance in annual, instalments of Rs 1000 plus 10% interest on the unpaid amount. How much the scooter will cost him.

GEOMETRIC PROGRESSION

21 Find the 5th term of the progression

$$1, \frac{(\sqrt{2}-1)}{2\sqrt{3}}, \left(\frac{3-2\sqrt{2}}{12}\right), \left(\frac{5\sqrt{2}-7}{24\sqrt{3}}\right),$$

22 Which term of the G.P. 5,10, 20,40,... is 5120 ?

If the 4th and 9th terms of a G.P. be 54 and 13122 respectively, find the G.P.

24 Which term of the G.P.:

(i) $\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{1}{4\sqrt{2}}, \ldots$ is $\frac{1}{512\sqrt{2}}$?

25 If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ $(x \neq 0)$, then show that a, b, c and d are in G.P.

26 The 4th term of a G.P. is square of its second term, and the first term is - 3. Find its 7^{th} term.

27 Find a G.P. for which the sum of first two terms is -4 and the fifth term is 4 times the third term.

28 The (m + n)th and (m - n)th terms of a G.P. are p and q respectively. Show that the mth and nth terms are $\sqrt{pq}$ and $P\left(\frac{q}{p}\right)^{m/2n}$ respectively.

29 If a, b, c are respectively the pth, qth and rth terms of a G.P., show that (q - r) log a + (r - p) log b + (p - q) log c = 0.

30 The first term of a G.P. is 1. The sum of the third term and fifth term is 90. Find the common ratio of the G.P.

31 The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

32 Find the sum to 7 terms of the sequence

$$\left(\frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}\right), \left(\frac{1}{5^4}+\frac{2}{5^5}+\frac{3}{5^6}\right), \left(\frac{1}{5^7}+\frac{2}{5^8}+\frac{3}{5^9}\right), \ldots.$$

33 The 4th and 7th terms of a G.P. are $\frac{1}{27}$ and $\frac{1}{729}$ respectively. Find the sum of n terms of the G.P.

34 Sum the series:

$x(x + y) + x^2(x^2 + y^2) + x^3(x^3 + y^3) + \ldots$ to n terms

35 In an increasing G.P., the sum of the first and the last term is 66, the product of the second and the last but one is 28 and the sum of the terms is 126. How many terms are there in the progression ?

36 If f is a function satisfying $f\{x + y) = f(x)\, f(y)$ for all x, y E N such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$, find the value of n.

37 A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8th set of letter is mailed.

38 Prove that: $6^{1/2} \cdot 6^{1/4} \cdot 6^{1/8} \ldots \infty = 6$.

39 Sum the following geometric series to infinity:

(i) $(\sqrt{2} + 1) + 1 + (\sqrt{2} - 1) + \ldots\infty$ (ii) $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{2^5} + \frac{1}{3^6} + \ldots\infty$

40 If $x = 1 + a + a^2 + \ldots\infty$, where $|a| < 1$ and $y = 1 + b + b^2 + \ldots\infty$, where $|b| < 1$. Prow that :

$$1 + ab + a^2b^2 + \ldots\infty = \frac{xy}{x + y - 1}$$

41 If $A = 1 + r^a + r^{2a} + \ldots +$ to ∞ and $B = 1 + r^b + r^{2b} + \ldots \infty$, prove that

$$r\left(\frac{A-1}{A}\right)^{1/a} = \left(\frac{B-1}{B}\right)^{1/b}$$

42 After striking a floor a certain ball rebounds $\left(\frac{4}{5}\right)$ th of the height from which it has fallen.

Find the total distance that it travels before coming to rest, if it is gently dropped from a height of 120 metres.

43 Find two positive numbers whose difference is 12. and whose A.M. exceeds the CM. by 2.

44 If the A.M. and G.M. between two numbers are in the ratio m : n, then prove that the

numbers are in the ratio $m+\sqrt{m^2-n^2}; m-\sqrt{m^2-n^2}$.

45 If a is the AM. of b and c and the two geometric means are G_1 and G_2, then prove that $G_1^3+G_2^3=2abc$

46 If the A.M. of two positive numbers a and b (a > b) is twice their geometric mean. Prove that: a : b = $(2+\sqrt{3}):(2-\sqrt{3})$.

47 If $\sum_{r=1}^{n} r = 55$, find $\sum_{r=1}^{n} r^3$.

48 Find the sum to n terms of the series :1 + 3 + 7 + 13 + 21 + ...

SUM OF SPECIAL SEQUNCES

49 Find the sum to n terms of the series :3 + 5 + 9 + 15 + 23 + ...

50 Find the sum to n terms of the series $1^2 + 3^2 + 5^2$ + ... to n terms.

51 The sequence N of natural numbers is divided into classes as follows:

1 2
3 4 5 6
7 8 9 10 11 12
.....
.....

Show that the sum of the numbers in nth row is $n(2n^2 + 1)$.

52 Show that: $\frac{1\times2^2+2\times3^2+...+n\times(n+1)^2}{1^2\times2+2^2\times3+...+n^2(n+1)}=\frac{3n+5}{3n+1}$.

53 Find the sum of the following series to n terms: $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+...$

54 Find the sum of the following series to n terms: $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{n.(n+1)}$

55 Find the sum to n terms of the series : $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+.....$

56 Find the sum of n terms of the series : $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+.....$

57 Find the sum to n terms of the series : $\frac{1}{1.6}+\frac{1}{6.11}+\frac{1}{11.14}+\frac{1}{14.19}+...+\frac{1}{(5n-4)(5n+1)}$