9.3 Geometric Progression (G.P.)

Definition

A progression is called a G.P. if the ratio of its each term to its previous term is always constant. This constant ratio is called its common ratio and it is generally denoted by *r*.

Example: The sequence 4, 12, 36, 108, ….. is a G.P., because $\frac{12}{4}=\frac{36}{12}=\frac{108}{36}=.....=3$, which is constant.

Clearly, this sequence is a G.P. with first term 4 and common ratio 3.

The sequence $\frac{1}{3},-\frac{1}{2},\frac{3}{4},-\frac{9}{8},....$ is a G.P. with first term $\frac{1}{3}$ and common ratio $\left(-\frac{1}{2}\right)\Big/\left(\frac{1}{3}\right)=-\frac{3}{2}$.

General term of a G.P.

(1) We know that, $a, ar, ar^2, ar^3, ...ar^{n-1}$ is a sequence of G.P.

Here, the first term is '*a*' and the common ratio is '*r*'.

The general term or n^{th} term of a G.P. is $T_n = ar^{n-1}$.

It should be noted that, $r=\frac{T_2}{T_1}=\frac{T_3}{T_2}=......$.

(2) **p^{th} term from the end of a finite G.P. :** If G.P. consists of '*n*' terms, p^{th} term from the end $=(n-p+1)^{th}$ term from the beginning $=ar^{n-p}$.

Also, the p^{th} term from the end of a G.P. with last term *l* and common ratio *r* is $l\left(\frac{1}{r}\right)^{n-1}$.

Selection of terms in a G.P.

(1) When the product is given, the following way is adopted in selecting certain number of terms :

Table 3

Number of terms	**Terms to be taken**
3	$\frac{a}{r}, a, ar$
4	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

Number of terms	Terms to be taken
5	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

(2) When the product is not given, then the following way is adopted in selection of terms

Table 4

Number of terms	Terms to be taken
3	a, ar, ar^2
4	a, ar, ar^2, ar^3
5	a, ar, ar^2, ar^3, ar^4

Sum of first ‘n’ terms of a G.P.

If a be the first term, r the common ratio, then sum S_n of first n terms of a G.P. is given by

$S_n = \frac{a(1-r^n)}{1-r}$ and $S_n = \frac{a-lr}{1-r}$, (when $|r| < 1$)

$S_n = \frac{a(r^n-1)}{r-1}$ and $S_n = \frac{lr-a}{r-1}$, (when $|r| > 1$)

$S_n = na$, (when $r = 1$)

Sum of infinite terms of a G.P.

(1) When $|r| < 1$, (or $-1 < r < 1$); $S_\infty = \frac{a}{1-r}$.

(2) If $r \geq 1$, then S_∞ doesn’t exist.

Geometric mean

If a, G, b are in G.P., then G is called G.M. between a and b.

(1) If $a, G_1, G_2, G_3, \ldots G_n, b$ are in G.P. then $G_1, G_2, G_3, \ldots G_n$ are called n G.M.’s between a and b.

(2) **Insertion of geometric means :** (i) **Single G.M. between *a* and *b* :** If a and b are two real numbers then single G.M. between a and $b = \sqrt{ab}$.

(ii) ***n* G.M.’s between *a* and *b* :** If $G_1, G_2, G_3, \ldots, G_n$ are n G.M.’s between a and b, then

$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, $G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$,

$G_3 = ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}$, ………………….., $G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$.

Properties of G.P.

(1) If all the terms of a G.P. be multiplied or divided by the same non-zero constant, then it remains a G.P., with the same common ratio.

(2) The reciprocal of the terms of a given G.P. form a G.P. with common ratio as reciprocal of the common ratio of the original G.P.

(3) If each term of a G.P. with common ratio *r* be raised to the same power *k*, the resulting sequence also forms a G.P. with common ratio r^k.

(4) In a finite G.P., the product of terms equidistant from the beginning and the end is always the same and is equal to the product of the first and last term. *i.e.*, if $a_1, a_2, a_3, \ldots\ldots a_n$ be in G.P.

Then $a_1 a_n = a_2 a_{n-1} = a_3 a_{n-2} = a_4 a_{n-3} = \ldots\ldots\ldots = a_r . a_{n-r+1}$

(5) If the terms of a given G.P. are chosen at regular intervals, then the new sequence so formed also forms a G.P.

(6) If $a_1, a_2, a_3, \ldots\ldots, a_n \ldots\ldots$ is a G.P. of non-zero, non-negative terms, then $\log a_1, \log a_2, \log a_3, \ldots\ldots \log a_n, \ldots\ldots$ is an A.P. and vice-versa.

(7) Three non-zero numbers *a*, *b*, *c* are in G.P., iff $b^2 = ac$.

(8) If first term of a G.P. of *n* terms is *a* and last term is *l*, then the product of all terms of the G.P. is $(al)^{n/2}$.

(9) If there be *n* quantities in G.P. whose common ratio is *r* and S_m denotes the sum of the first *m* terms, then the sum of their product taken two by two is $\frac{r}{r+1} S_n S_{n-1}$.

(10) If $a^{x_1}, a^{x_2}, a^{x_3}, \ldots., a^{x_n}$ are in G.P., then $x_1, x_2, x_3, \ldots., x_n$ will be are in A.P. ,