9.6 FEW IMPORTANT RESULTS

- If T_k and T_p of any A.P. are given, then formula for obtaining T_n is $\frac{T_n - T_k}{n-k} = \frac{T_p - T_k}{p-k}$.
- If $pT_p = qT_q$ of an *A.P.*, then $T_{p+q} = 0$.
- If p^{th} term of an A.P. is q and the q^{th} term is p, then $T_{p+q} = 0$ and $T_n = p + q - n$.
- If the p^{th} term of an A.P. is $\frac{1}{q}$ and the q^{th} term is $\frac{1}{p}$, then its pq^{th} term is 1.
- The common difference of an A.P is given by $d = S_2 - 2S_1$ where S_2 is the sum of first two terms and S_1 is the sum of first term or the first term.
- If sum of n terms S_n is given then general term $T_n = S_n - S_{n-1}$, where S_{n-1} is sum of $(n-1)$ terms of A.P.
- Sum of n terms of an A.P. is of the form $An^2 + Bn$ i.e. a quadratic expression in n, in such case, common difference is twice the coefficient of n^2 *i.e.*, $2A$.
- Some standard results
  - Sum of first n odd natural numbers

    $$= 1 + 3 + 5 + ... + (2n-1) = \sum_{r=1}^{n} (2r-1) = n^2.$$

  - Sum of first n even natural numbers

    $$= 2 + 4 + 6 + ... + 2n = \sum_{r=1}^{n} 2r = n(n+1).$$

- • If for an A.P. sum of p terms is q and sum of q terms is p, then sum of $(p + q)$ terms is $\{-(p + q)\}$.
  - If for an A.P., sum of p terms is equal to sum of q terms, then sum of $(p + q)$ terms is zero.
  - If the p^{th} term of an A.P. is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, then sum of pq terms is given by $S_{pq} = \frac{1}{2}(pq+1)$.
- Sum of n A.M.'s between a and b is equal to n times the single A.M. between a and b. i.e. $A_1 + A_2 + A_3 + ... + A_n = n\left(\frac{a+b}{2}\right)$.
- If A_1 and A_2 are two A.M.'s between two numbers a and b, then $A_1 = \frac{1}{3}(2a+b), A_2 = \frac{1}{3}(a+2b)$.
- Between two numbers, $\frac{\text{Sum of } m \text{ A.M.'s}}{\text{Sum of } n \text{ A.M.'s}} = \frac{m}{n}$.
- If number of terms in any series is odd, then only one middle term exists which is

$\left(\frac{n+1}{2}\right)^{th}$ term.

- If number of terms in any series is even then there are two middle terms, which are given by $\left(\frac{n}{2}\right)^{th}$ and $\left\{\left(\frac{n}{2}\right)+1\right\}^{th}$ term.

- If T_k and T_p of any G.P. are given, then formula for obtaining T_n is $\left(\frac{T_n}{T_k}\right)^{\frac{1}{n-k}} = \left(\frac{T_p}{T_k}\right)^{\frac{1}{p-k}}$.

- If *a, b, c* are in G.P. then

$\Rightarrow \frac{b}{a} = \frac{c}{b} \Rightarrow \frac{a+b}{a-b} = \frac{b+c}{b-c}$ or $\frac{a-b}{b-c} = \frac{a}{b}$ or $\frac{a+b}{b+c} = \frac{a}{b}$.

- Let the first term of a *G.P* be positive, then if $r > 1$, then it is an increasing G.P., but if r is positive and less than 1, *i.e.* $0 < r < 1$, then it is a decreasing G.P.

- Let the first term of a G.P. be negative, then if $r > 1$, *then it* is a decreasing G.P., but if $0 < r < 1$, then it is an increasing G.P.

- If *a, b, c, d,…* are in G.P., then they are also in continued proportion i.e. $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = ... = \frac{1}{r}$.

- Product of *n* G.M.'s between *a* and *b* is equal to n^{th} power of single geometric mean between *a* and *b*,

  i.e. $G_1 G_2 G_3 ...... G_n = (\sqrt{ab})^n$.

- G.M. of $a_1 a_2 a_3 ...... a_n$ is $(a_1 a_2 a_3 ..... a_n)^{1/n}$.

- If G_1 and G_2 are two G.M.'s between two numbers *a* and *b* is $G_1 = (a^2 b)^{1/3}, G_2 = (ab^2)^{1/3}$.